Factoring Methods Covered...

- Greatest Common Factor
- Simple Trinomials
- Hard Trinomials
- Perfect Square Trinomials
- Difference of Squares
- Sum & Diff. of Cubes

New Factoring . . .

Sum & Difference of Cubes

$x^3 - y^3$

$(x - y)(x^2 + xy + y^2)$

cube Root of 1st; 1st; 2nd; square 1st; Product of 1st x 2nd x (-1) reverse sign; square the 2nd

$(a^6 + b^9)$

$(a^2 + b^3)(a^4 - a^2b^3 + b^6)$

$(a^2)^2$, $(b^3)^2$

ex.

$\sqrt[3]{x^{27}} = (x^{27})^{\frac{1}{3}}$

① $8x^{27} - 27y^{12}$

$(2x^9 - 3y^4)(4x^{18} + 6x^9y^4 + 9y^8)$

② $w^{12} - 1$

Cubes

$(w^4 - 1)(w^8 + w^4 + 1)$

$(w^2 - 1)(w^2 + 1)(w^8 + w^4 + 1)$

$(w - 1)(w + 1)(w^2 + 1)(w^8 + w^4 + 1)$

Start with Diff. of Squares

$(w^6 - 1)(w^6 + 1)$

$(w^3 - 1)(w^3 + 1)(w^6 + 1)$

$(w - 1)(w^2 + w + 1)(w + 1)(w^2 - w + 1)(w^2 + 1)(w^4 - w^2 + 1)$

Challenge!!!!

Factor Completely:

$$\boxed{\underline{x^4 - 10x^2y + 25y^2} - 4x^2 - 4xy^2 - y^4}$$

$$(x^4 - 10x^2y + 25y^2) - (4x^2 + 4xy^2 + y^4)$$

$$a^2 - b^2 \leftarrow (x^2 - 5y)^2 - (2x + y^2)^2$$

$$[(x^2 - 5y) - (2x + y^2)][(x^2 - 5y) + (2x + y^2)]$$

$$\underline{(x^2 - y^2 - 5y - 2x)}\,\underline{(x^2 + y^2 - 5y + 2x)}$$

Factor:

$$27w^{27} \pm 8y^{9}$$

. . → New factoring Technique!!!

⇒ Sum & Difference of Cubes

$$(3w^{9} \pm 2y^{3})(9w^{18} \mp 6w^{9}y^{3} + 4y^{6})$$

Review of Long Division

```
   261
3)785
  6
  --
  18
  18
  --
    5
    3
    -
    2
```

$3(261)+2=785$

```
   2345.
4)9382.000
  8
  --
  13
  12
  --
   18
   16
   --
    22
    20
    --
     2
```

Long Division of Polynomials

Example... $(x^3 + x^2 - 3x - 1) \div (x+2)$

Quotient

Remainder

Write As... $x + 2 \overline{) x^3 + x^2 - 3x - 1}$ Dividend

Divisor

$$\begin{array}{r} x^2 - x - 1 \\ x+2 \overline{) x^3 + x^2 - 3x - 1} \\ \underline{x^3 + 2x^2 } \\ -x^2 - 3x \\ \underline{-x^2 - 2x } \\ -x - 1 \\ \underline{-x - 2} \\ \text{Rem} = 1 \end{array}$$

$(x+2)(x^2-x-1) + 1 =$

Check... Divisor x Quotient + Remainder = Dividend

Example #2... $x - 2 \overline{) x^3 - 4x^2 + x + 6}$

$$\begin{array}{r} x^2 - 2x - 3 \\ x-2 \overline{) x^3 - 4x^2 + x + 6} \\ \underline{x^3 - 2x^2 } \\ -2x^2 + x \\ \underline{-2x^2 + 4x } \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

$(x-2)(x^2-2x-3) =$

$(x-2)(x-3)(x+1)$

Ex. Divide using long division: $x^2 + 0x - 1$

$$\left(2x^3 - x^2 + 5x - 3\right) \div \left(x^2 - 1\right)$$

$$\begin{array}{r|l} & 2x - 1 \\ x^2 + 0x - 1 & \overline{2x^3 - x^2 + 5x - 3} \\ & 2x^3 + 0x^2 - 2x \\ & \overline{\quad -x^2 + 7x - 3} \\ & \quad -x^2 - 0x + 1 \\ & \overline{\qquad 7x - 4} \end{array}$$

$(2x-1)(x^2-1) + 7x - 4 =$

$$
\begin{array}{r}
2x - 1 \\
x^2 - 1 \enclose{longdiv}{2x^3 - x^2 + 5x - 3} \\
\underline{2x^3 - 2x} \\
-x^2 + 7x \\
\underline{-x^2 + 1} \\
7x - 4
\end{array}
$$

$$x^3 - 2$$

$$x^{3} + 2 \overline{) x^6 - x + 3}$$

$$x^6 + 2x^3$$

$$(x^3+2)(x^3-2) + -x+7 \qquad -2x^3 - x$$

$$-(x-7) \qquad -2x^3 - 4$$

$$-x^1 + 7$$

$$\begin{array}{r|l} & x^4 - 4x \\ \hline x^4+2x & x^8 - 2x^5 + x - 1 \\ & \underline{x^8 + 2x^5} \\ & \quad -4x^5 + x \\ & \quad \underline{-4x^5 - 8x^2} \\ & \qquad 8x^2 + x - 1 \end{array}$$

Check-Up:

$$x-2\;\overline{)\,3x^4-2x^3+x^2-5x+4}$$

Quotient: $3x^3+4x^2+9x+13$

$3x^4-6x^3$

$4x^3+x^2$

$4x^3-8x^2$

$9x^2-5x$

$9x^2-18x$

$13x+4$

$13x-26$

Rem $=30$

-2	3	-2	1	-5	4
		-6	-8	18	-26
	3	4	9	13	30

$$x^3+2\;\overline{)\,x^8-x^7+x-1}$$

Quotient: $x^5-x^4-2x^2+2x$

x^8+2x^5

$-x^7-2x^5$

$-x^7-2x^4$

$-2x^5+2x^4$

$-2x^5-4x^2$

$2x^4+4x^2$

$2x^4+4x$

Rem $=4x^2-3x-1$

Synthetic Division

$$\left(1x^3 + 1x^2 - 3x - 1\right) \div \left(x + \underset{-2}{2}\right)$$

can't do synthetic division if power or coefficient is greater than 1!

1. Write coefficients **only** in upside-down symbol.
2. Drop variable of the divisor, and put on the left.
3. Carry down the first number (represents leading coefficient).
4. Multiply by number outside, carry result up into next column, and subtract down the column.
5. Translate coefficients into polynomial with the last number being the remainder.

$$\begin{array}{r|rrrr}
 & x^3 & x^2 & x^1 & x^0 \\
2 & 1 & 1 & -3 & -1 \\
\hline
 & & 2 & -2 & -2 \\
\hline
 & 1 & -1 & -1 & 1
\end{array}$$

$$\underbrace{1 \quad -1 \quad -1}_{\text{Quotient}} \qquad \underbrace{1}_{\text{Remainder}}$$

$(x^2 - x - 1)$ $\qquad R = 1$

$$\begin{array}{r}
x^2 - 1x - 1 \\
x+2 \overline{\big) x^3 + x^2 - 3x - 1} \\
\underline{x^3 + 2x^2 \qquad\qquad} \\
-1x^2 - 3x \quad \\
\underline{-x^2 - 2x \quad} \\
-1x - 1 \\
\underline{-x - 2} \\
1
\end{array}$$

Ex. Divide $\left(-3x^2-10x+2x^3+5\right) \div (x-3)$
using long *and* synthetic division.

* Dividend MUST be in descending order of exponents

$\longrightarrow 2x^3-3x^2-10x+5$

$$\begin{array}{r|rrrr} -3 & 2 & -3 & -10 & 5 \\ & \downarrow & -6 & -9 & 3 \\ \hline & 2 & 3 & -1 & 2 \end{array}$$

$=\left(2x^2+3x-1\right)$ Rem $=2$

Ex. Divide $\left(x^3+9x+10\right) \div \left(x+2\right)$

using ~~long~~ *and* synthetic division.

$$
\begin{array}{r|rrrr}
 & x^3 & x^2 & x & x^0 \\
2 & 1 & 0 & 9 & 10 \\
 & & 2 & -4 & 26 \\
\hline
 & 1 & -2 & 13 & -16
\end{array}
$$

$= \left(x^2-2x+13\right)$ Rem $= -16$

Practice

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Remainder Theorem

Remainder Theorem

If f(x) is divided by (x-n), then the remainder is f(n).

Ex. What is the remainder when $\left(2x^3 - 4x^2 + 3x - 6\right)$ is divided by $\left(x+2\right)$?

$$\begin{array}{r|rrrr} 2 & 2 & -4 & 3 & -6 \\ & & 4 & -16 & 38 \\ \hline & 2 & -8 & 19 & -44 \end{array}$$

Synthetic

Remainder Theorem

$(3x+4)$

$(x+\frac{4}{3})$

$f(-2) = 2(-2)^3 - 4(-2)^2 + 3(-2) - 6$

$= -16 - 16 - 6 - 6$

$= -44$

12

4 3

What is the remainder when $\left(3x^4 - x^3 - 4x^2 + x - 5\right)$ is divided by $\left(x-1\right)$?

Sub. $x=1$

$3 - 1 - 4 + 1 - 5$

$= -6$