Warm Up

It is spring training and skipper Cito Gaston has 9 outfielders available to fill 3 spots in the lineup. How many different outfields would be possible?

$${}_9P_3 = 504 \text{ (Position Matters)} \qquad {}_9C_3 = 84$$

How many different arrangements could be made using the digits "24553448"?

$$\frac{{}_8P_8}{2!\,3!} = 3360$$

A game of chance requires a player to roll a pair of six-sided dice. The player wins if both dice show odd numbers or if both dice show the same number. What is the probability a player will win?

Win ⇒ Both Odd or Doubles

Counted Twice

$$P(W) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \frac{6}{36} - \frac{3}{36}$$

$$= \frac{1}{4} + \frac{1}{6}$$

$$= \frac{3+2}{12}$$

$$= \frac{5}{12} - \frac{3}{36}$$

$$= \frac{15-3}{36} = \frac{12}{36} = \frac{1}{3}$$

Return to the Binomial Theorem...

Binomial	Pascal's Triangle in Binomial Expansion	Row
$(x+y)^0$	1	1
$(x+y)^1$	$1x + 1y$	2
$(x+y)^2$	$1x^2 + 2xy + 1y^2$	3
$(x+y)^3$	$1x^3 + 3x^2y + 3xy^2 + 1y^3$	4
$(x+y)^4$	$1x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + 1y^4$	5

$(a+b)^{14}$

Have a look at this neat connection...

Pascal's Triangle	Combinations
1	${}_0C_0$
1 1	${}_1C_0$ ${}_1C_1$
1 2 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$
1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$
1 4 6 4 1	${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$
1 5 10 10 5 1	${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$

You can use the **binomial theorem** to expand any power of a binomial expression.

$$(x+y)^n = {}_nC_0(x)^n(y)^0 + {}_nC_1(x)^{n-1}(y)^1 + {}_nC_2(x)^{n-2}(y)^2 + \cdots + {}_nC_{n-1}(x)^1(y)^{n-1} + {}_nC_n(x)^0(y)^n$$

$(2x+3y^3)^9 \Rightarrow$ How many terms?? 10 terms

$\Rightarrow$ What is the 7th term?

$${}_9C_6 \, (2x)^{3}(3y^3)^6$$

$n-6$

$84(2x)^3(3y^3)^6$

$=(84 \times 8 \times 729)\, x^3y^{18}$

$= \underline{489\,888x^3y^{18}}$

ex. $(3w^3 - y^2)^{18}$

What is numerical coefficient if variable part is $w^{21}y^{22}$?

$= {}_{18}C_{11}(3w^3)^7(-y^2)^{11}$

$31824 \times 2187 \times -1$

$-69\,599\,088\,w^{21}y^{22}$

$$(-2x^4+3y^8)^9$$

① What is 5th term?

② What is numerical coefficient for term with $x^{16}y^{40}$ as the variable part?

① $${}_9C_4(-2x^4)^5(3y^8)^4 = 126 \times -32 \times 81 x^{20}y^{32}$$
$$= -326592x^{20}y^{32}$$

② $${}_9C_5(-2x^4)^4(3y^8)^5$$

Expand: $(-2x^3+y^5)^6$

Pascal's Δ

$$\begin{array}{ccccccccccccc}
&&&&&&1&&&&&&\\
&&&&&1&&1&&&&&\\
&&&&1&&2&&1&&&&\\
&&&1&&3&&3&&1&&&\\
&&1&&4&&6&&4&&1&&\\
&1&&5&&10&&10&&5&&1&\\
\rightarrow 1&&6&&15&&20&&15&&6&&1
\end{array}$$

coefficients

OR

$1\times64\times1$ $\quad$ $6\times-32\times1$ $\quad$ $15\times16\times1$

$${}_6C_0(-2x^3)^6(y^5)^0+{}_6C_1(-2x^3)^5(y^5)^1+{}_6C_2(-2x^3)^4(y^5)^2$$

$20\times-8\times1$ $\quad$ $15\times4\times1$ $\quad$ $6\times-2\times1$

$$+{}_6C_3(-2x^3)^3(y^5)^3+{}_6C_4(-2x^3)^2(y^5)^4+{}_6C_5(-2x^3)^1(y^5)^5$$

$1\times1\times1$

$$+{}_6C_6(-2x^3)^0(y^5)^6$$

$$=64x^{18}-192x^{15}y^5+240x^{12}y^{10}-160x^9y^{15}$$
$$+60x^6y^{20}-12x^3y^{25}+y^{30}$$

Practice ....

Pg. 543
#17,18,19,20

Pg. 547
#16,17

Polynomials Review

- Factoring Techniques
- Dividing Polynomials
- Remainder Theorem
- Factor Theorem
- Sum and Difference of Cubes
- Solving Polynomial Equations
- Permutations and Combinations
- Binomial Theorem

Review-Polynomials.pdf

Review

Pg. 153-154

4, 5, 6, 7, 8, 10

Practice Test

Pg. 155

1, 2, 3, 4, 6, 7