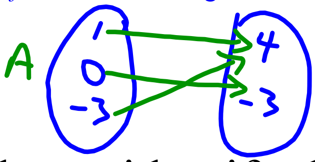
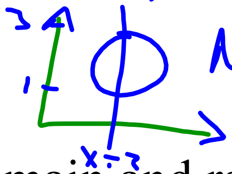
Functions

What is a function?

A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$.





Not a function

How do we identify the domain and range of a function?

- First must know what these terms mean...define each.
- Must know how to indicate domain and range using correct notation (Set and Bracket)

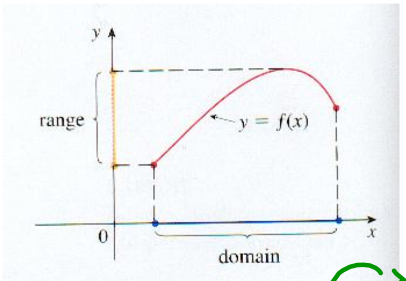
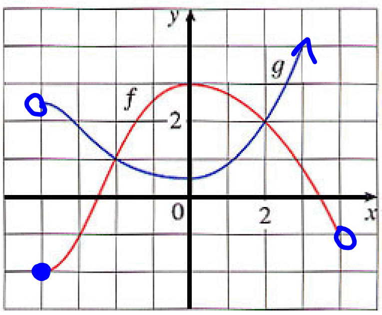
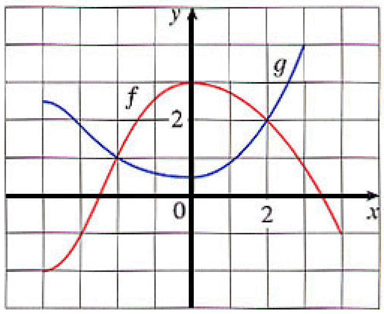


FIGURE 5

$g: \begin{cases} D: \{x \mid x > -4, x \in R\} & (-4, \infty) \\ R: \{y \mid y \geq \frac{1}{2}, y \in R\} & [\frac{1}{2}, \infty) \end{cases}$

Examples:





F:

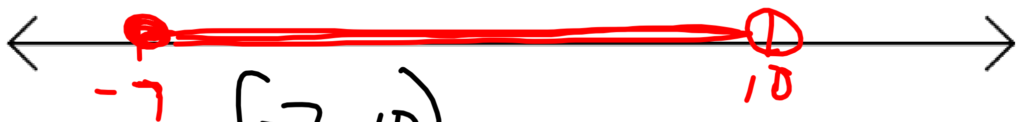
$D: \{x \mid -4 \leq x < 4, x \in R\}$ $[-4, 4)$

such that

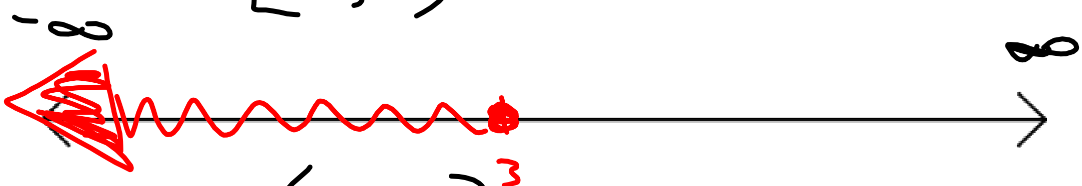
$R: \{y \mid -2 \leq y \leq 3, y \in R\}$ $[-2, 3]$

Bracket Notation

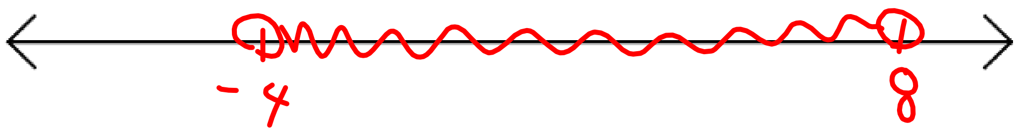
Notation used to describe "SETS" of numbers...



$[-7, 10)$



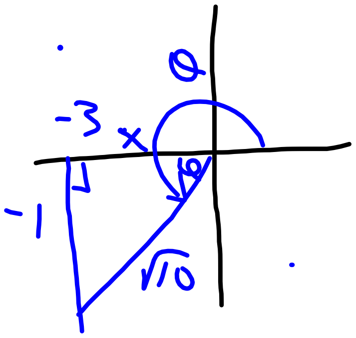
$(-\infty, 3]$



$(-4, 8)$

Check-Up

1. If $\sin\theta = -\frac{1}{\sqrt{10}}$ and $\cos\theta < 0$ find $\tan\theta$



$x^2 = (\sqrt{10})^2 - (1)^2$

$x^2 = 10 - 1$

$x^2 = 9$

$x = 3$

$\tan\theta = \frac{1}{3}$

2. Determine the domain and range of the quadratic $f(x) = -5x^2 + 10x - 3$.

D: $x \in R$

$f(x) = \overset{-5}{-5}(x^2 - 2x + 1) - 3 + 5$

$= -5(x-1)^2 + 2$

$\Rightarrow V(1,2)$

$-\infty$ 2 Opens Down

$(-\infty, 2]$

Smallest first

$y \geq 2$

$-\infty$

$f(x) = 3x^2 - 18x + 2$

$= 3(x^2 - 6x + 9 \quad) + 2 - 27$

$= 3(x^2 - 6x + 9) - 27 + 2$

Domain & Range: $\longrightarrow x \in R$

"y"

ex. $f(x) = \sqrt[30]{x+7} \longrightarrow y \in R$

D: $x+7 \geq 0$

$x \geq -7$ R: $y \geq 0$

"y"

$f(x) = \frac{3}{2x+4} = \frac{0}{\#} = 0$

D: $2x+4 \neq 0$

$\frac{2x}{2} \neq \frac{-4}{2}$

$x \neq -2$

$\boxed{x \in R, x \neq -2}$

Range: $y \in R, y \neq 0$

$(-\infty, 0) \cup (0, \infty)$

"OR"

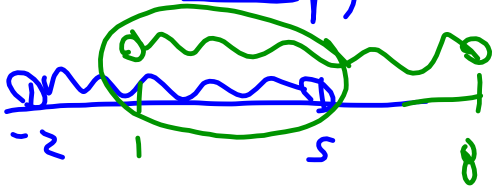
$(-\infty, -2) \cup (-2, \infty)$

"In Union With"

$(-2, 5) \cap (1, 8)$

"Intersection of"

(overlap)



$(1, 5)$

Check for Understanding...

Select the best response for each of the following:

1. Find the domain of $f(x) = \sqrt{2x+3}$.

a) $[0, \infty)$ b) $(0, \infty)$ c) $[-\frac{3}{2}, \infty)$
d) $(-\frac{3}{2}, \infty)$ e) $[0, \frac{3}{2})$

$2x+3 \geq 0$
$x \geq -\frac{3}{2}$

2. Find the range of the function $y = \frac{1}{x-3}$.

a) $(3, \infty)$ b) $(-\infty, 3)$
c) $(-\infty, \frac{1}{3}),\ (\frac{1}{3}, \infty)$ d) $(-\infty, 3),\ (3, \infty)$
e) $(-\infty, 0) \cup (0, \infty)$

3. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6 (B) -3 (C) -1 (D) 2

(E) It cannot be determined from the information given.

4. Solve: $x^2 - x > 12$

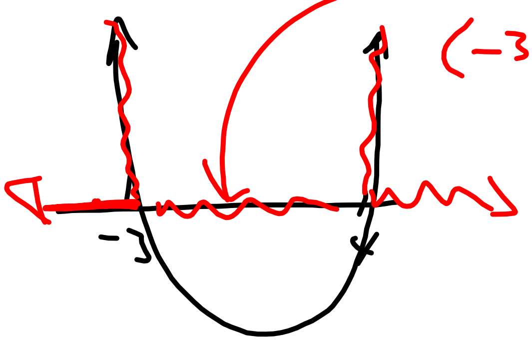
a) $x < -6$ or $x > 1$ b) $x < -3$ or $x > 4$
c) $x < -2$ or $x > 3$ d) $-6 < x < 1$
e) $-2 < x < 3$

positive

$x^2 - x - 12 > 0$

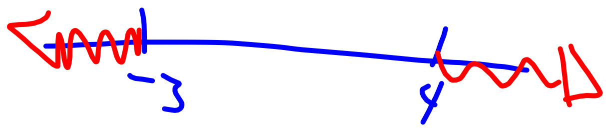
$(x-4)(x+3) = 0$

$x = 4, -3$



$> -3 < x < 4$
$(-3, 4)$

$x < -3$ or $x > 4$



3. If $f(x) = 2x^3 + Ax^2 + Bx - 5$ and if $f(2) = 3$ and $f(-2) = -37$, what is the value of $A + B$?

(A) -6 (B) -3 (C) -1 (D) 2

(E) It cannot be determined from the information given.

$$2(2)^3 + A(2)^2 + B(2) - 5 = 3$$
$$16 + 4A + 2B - 5 = 3$$
$$\boxed{4A + 2B = -8}$$

$$2(-2)^3 + A(-2)^2 + B(-2) - 5 = -37$$
$$-16 + 4A - 2B - 5 = -37$$
$$\boxed{4A - 2B = -16}$$

$$+ \begin{cases} 4A + 2B = -8 \\ 4A - 2B = -16 \end{cases}$$
$$\frac{8A}{8} = \frac{-24}{8}$$
$$A = -3$$

$$4(-3) + 2B = -8$$
$$\frac{2B}{2} = \frac{4}{2}$$
$$B = 2$$

$$A + B = -3 + 2$$
$$= -1$$

Function Notation

- Must understand the notation associated with determing the values of functions

I. From a graph

$f(1)=7$

"Determine the y-value, when x=1"

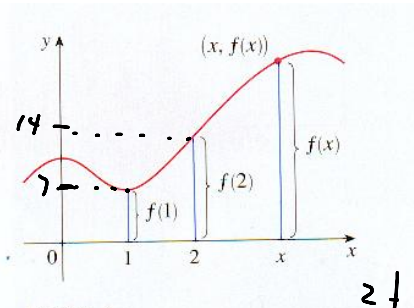
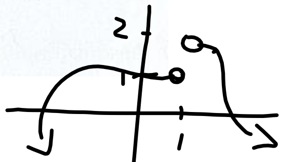


FIGURE 4



$f(1)=$ DNE

Does Not Exist

II. From a table of values

$f(13)=-8$

x	$f(x)$
−3	7
10	4
13	−8

I. From an explicit formula (Equation)

$=-2(-3)^2+5(-3)-3$

$=-18-15-3$

$=-36$

$=-2(\cancel{8})^2+5(\cancel{8})-3$

$f(x) = -2x^2 + 5x - 3$ ← Explicit formula !

$f(-3) = ?$ $f(\$) = ?$

$f(2 - h) = ?$

$(2-h)(2-h)$

$=-2(2-h)^2+5(2-h)-3$

$=-2(4-4h+h^2)+10-5h-3$

$=-8+8h-2h^2-5h+7$

$f(2-h)\boxed{=-2h^2+3h-1}=0$

$(f\circ g)2$

$f[g(2)]$