Pre-Calculus A 120 **Name:**_____
Midterm **November 2014**

Show all work for each of the following in the space provided. **[TOTAL: 75 Marks]**

1. The base function $f(x)=x^3$ is reflected in the *x*-axis, stretched horizontally by a factor of 5, stretched vertically by a factor of 2 and translated 3 units to the right and 4 units down.

   (a) Write an equation using function notation $g(x)=a\,f[b(x+c)]+d$ of the transformed function *g*(*x*). [4]

   (b) Write a mapping rule that would transform the graph of *f*(*x*) into the graph of *g*(*x*). [4]

   (c) Given that the ordered pair (-2 , -8) lies on the graph of *f*(*x*), what are the coordinates of this point on the graph of *g*(*x*)? [2]

   (d) Without using the ordered pair from part (c), determine the coordinates of any ordered pair that would fall on the on the **inverse** of *g*(*x*)? **(Work must clearly be shown!!)** [3]

2. Given that $f(x)=-2x$, $g(x)=1-x^2$, *and* $w(x)=\sqrt{x+12}$ …

   (a) Evaluate $(f\circ g\circ w)(-3)$. [3]

   (b) Determine an expression in simplest form for $(f-g)(b^2)+3[f(b^4)]^2$ [5]

3. Complete the chart and sketch **one** full cycle for the functions in the space provided below. Be sure to **clearly label** and scale both axes on your graphs. [15]

$$4(y+1) = -16\sin\left(3\theta - \frac{\pi}{2}\right) + 8$$

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	
MAPPING NOTATION	

Sketch→

4. The picture below depicts a windmill found at the Atlantic Wind Test Site located at North Cape, Prince Edward Island. The three blades on this particular windmill are each 12 m long, and it is stationed atop a 44 m tower. A scientist starts a stopwatch and begins recording the height above the ground of the tip of one of the blades. He observes that at 1.8 seconds the blade is closest to the ground, and that 1.5 seconds later the blade reaches it's highest point.

(a) Draw a sketch that depicts the height above the ground of the tip of one of the blades at any time in seconds. ***Be sure the graph is clearly labeled and scaled***! [3]

(b) Determine a function *h(t)* that would determine the height above the ground in metres of the tip of one of the blades at any time in seconds. [4]

(c) Determine the height above the ground of this particular blade 3 minutes and 29 seconds after the stopwatch was initially started. [2]

(d) Determine the angular velocity in radians per second of the windmill. [2]

(e) Determine the distance that the tip of one of the blades would travel after 50 seconds has elapsed. [3]

(f) Determine the **first three instances** that this particular blade being examined by the scientist would have been located at a height of 38 m above the ground. [5]

5. Evaluate the following expression without using a calculator: (***Sketch must be provided for each angle***) [12]

$$\sin\left(-\frac{2\pi}{3}\right)\sec\left(\frac{107\pi}{6}\right)-5\cos(-33\pi)-2\csc^2\left(\frac{19\pi}{4}\right)+\cot\left(\frac{37\pi}{2}\right)$$

6. The University of Calgary's Institute for Space Research is leading a project to launch Cassiope, a hybrid space satellite. Cassiope will follow a path that may be modelled by the function $h(t)=350\sin[24\pi(t-25)]+400$, where *h* is the height, in kilometres, of the satellite above Earth and *t* is the time, in days after being launched.

(a) What are the minimum and maximum heights that this satellite will orbit above the earth? [2]

(c) Determine any two instances after being launched into orbit that this satellite will be situated 500 km above the earth. ***(Answers must be correct to the nearest hour)*** [6]