Series

Summation Notation:

The capital Greek letter sigma $\sum$ is used to provide a shorthand notation for a summation.

In summation notation, the sum of the terms of the sequence $\{a_1, a_2, a_3, \ldots, a_n\}$

is denoted $\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \ldots + a_n$

which is read "the sum of a_k from $k = 1$ to n"

Sigma notation is actually more widely used than the above definition suggests.

See if you can evaluate each of the following summations:

$$\sum_{k=1}^{5} 3k = 3(1)+3(2)+3(3)+3(4)+3(5)$$
$$= 3+6+9+12+15$$
$$= 45$$

$$\sum_{k=4}^{9} (k-2)$$
$$= (4-2)+(5-2)+(6-2)+(7-2)+(8-2)+(9-2)$$
$$= 2+3+4+5+6+7$$
$$= 27$$

$$\sum_{k=1}^{5} (k^2-3k)$$
$$= [(1)^2-3(1)] + [(2)^2-3(2)] + [(3)^2-3(3)] + [(4)^2-3(4)] + [(5)^2-3(5)]$$
$$= -2 + -2 + 0 + 4 + 10$$
$$= 10$$

Arithmetic Series:

- The summation of the terms of an arithmetic sequence

Formula $\dashrightarrow S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Let's derive this formula that can be used to determine the sum of "n" terms in any arithmetic series...

$$S = a + (a+d) + (a+2d) + (a+3d) + \ldots + [a+(n-2)d] + [a+(n-1)d]$$

$$S = a + (n-1)d + a + (n-2)d + \cdots + a + d + a$$

$$2S = \underbrace{2a + (n-1)d + 2a + (n-1)d + \cdots + 2a + (n-1)d + 2a + (n-1)d}_{n \text{ of these}}$$

$$\frac{2S}{2} = \frac{n[2a + (n-1)d]}{2}$$

$$\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$$

ex.

Find S_{131} for $-3+7+17+\ldots$

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{131}=\frac{\overset{n}{131}}{2}\left[2(\underset{a}{-3})+(\overset{n}{131}-1)(\overset{d}{10})\right]$$

$$=\underline{84757}$$

Example:

Evaluate the following summation:

$n=159$

$$-2+7+16+25+34+\ldots+1420=$$

$t_n = a+(n-1)d$

$1420 = -2+(n-1)(9)$

$\frac{1422}{9} = \frac{(n-1)9}{9}$

$158 = n-1$

$159 = n$

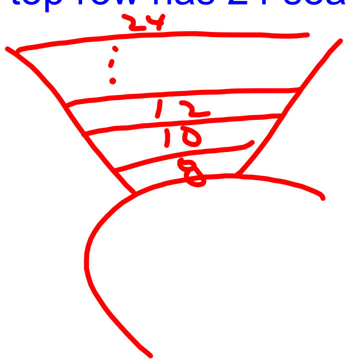
$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$S_{159} = \frac{159}{2}\left[2(-2)+(158)(9)\right]$

$= 112731$

Example:

A corner section of a stadium has 8 seats along the front row. Each successive row has two more seats than the row preceding it. If the top row has 24 seats, how many seats are in the entire section?



$8+10+12+\cdots+24$

$24 = 8+(n-1)(2)$

$16 = (n-1)(2)$

$8 = n-1$

$9 = n$

$S_9 = \frac{9}{2}\left[2(8)+(8)(2)\right]$

$S_9 = 144$ seats

Geometric Series:

- The summation of the terms of a geometric sequence

Formula → $S_n = \frac{a(1-r^n)}{1-r}$ OR $S_n = \frac{a(r^n-1)}{r-1}$

Let's derive this formula that can be used to determine the sum of "n" terms in any geometric series...

$$S = a + \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \ldots + \cancel{ar^{n-2}} + \cancel{ar^{n-1}}$$

- Multiply both sides of this summation by r and then subtract the two equations

$$Sr = 0 + \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \cdots + \cancel{ar^{n-2}} + \cancel{ar^{n-1}} + ar^n$$

$$S - Sr = a - ar^n$$

$$S(1-r) = a(1-r^n)$$

$$\boxed{S_n = \frac{a(1-r^n)}{1-r}} *$$

Find S_{14} of the series

$-3 + -6 + -12 + \dots$

$a = -3$
$r = 2$
$n = 14$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_{14} = \frac{-3((2)^{14} - 1)}{2 - 1}$$

$$S_{14} = \underline{-49149}$$

Example:

Find the sum of each of the following series:

(a) $S = -3 - 6 - 12 - 24 - \ldots - 196608$

$-196608 = -3(2)^{n-1}$

$65536 = 2^{n-1} \rightarrow \log_2 65536$

$2^{16} = 2^{n-1}$

$16 = n-1$

$17 = n$

$S_{17} = \dfrac{(-3)(2^{17}-1)}{2-1}$

$S_{17} = -393213$

(b) $S = 5 - 10 + 20 - 40 + \ldots + 5242880$

Pg. 27
4, 5, 6

Pg. 53
3, 4, 5

Attachments

NOTES - Standard to Vertex Form.pdf