SKILLS SUMMARY

Skill	Description	Example
Classify numbers. [4.1, 4.2]	To determine whether a number is rational or irrational, write the number in decimal form. ■ Repeating and terminating decimals are rational. ■ Non-repeating, non-terminating decimals are irrational.	Rational numbers: $2, 0, -3, 3.75, 0.0\overline{1}, \frac{3}{5}, -\frac{10}{7}$ Irrational numbers: $\sqrt{3}, \pi$
Simplify radicals. [4.3]	To simplify a square root: **1.** Write the radicand as a product of its greatest perfect square factor and another number. **2.** Take the square root of the perfect square factor. A similar procedure applies for cube roots and higher roots.	$\sqrt{200} = \sqrt{100 \cdot 2}$ $= \sqrt{100} \cdot \sqrt{2}$ $= 10\sqrt{2}$ $\sqrt[3]{200} = \sqrt[3]{8 \cdot 25}$ $= \sqrt[3]{8} \cdot \sqrt[3]{25}$ $= 2\sqrt[3]{25}$
Evaluate powers. [4.4, 4.5]	To evaluate powers without using a calculator: **1.** Rewrite a power with a negative exponent as a power with a positive exponent. **2.** Represent powers with fractional exponents as radicals. **3.** Use mental math to evaluate the powers and/or simplify the roots.	$64^{-\frac{2}{3}} = \frac{1}{64^{\frac{2}{3}}}$ $= \frac{1}{(\sqrt[3]{64})^2}$ $= \frac{1}{4^2}$ $= \frac{1}{16}$
Apply the exponent laws to simplify expressions. [4.6]	To simplify expressions involving powers: **1.** Remove brackets by applying the exponent laws for products of powers, quotients of powers, or powers of powers. **2.** Write the simplest expression using positive exponents.	$\left(\frac{(xy^2)^3}{x^5y}\right)^{-4} = \left(\frac{x^3y^6}{x^5y}\right)^{-4}$ $= \left(\frac{x^5y^1}{x^3y^6}\right)^4$ $= (x^{5-3}y^{1-6})^4$ $= (x^2y^{-5})^4$ $= x^8y^{-20}$ $= \frac{x^8}{y^{20}}$

REVIEW

4.1

1. Evaluate each radical. Why do you not need a calculator?

a) $\sqrt[3]{1000}$ **b)** $\sqrt{0.81}$

c) $\sqrt[6]{64}$ **d)** $\sqrt[4]{\frac{81}{625}}$

2. Explain, using examples, the meaning of the index of a radical.

3. Estimate the value of each radical to 1 decimal place. What strategies can you use?

a) $\sqrt{11}$ **b)** $\sqrt[3]{-12}$ **c)** $\sqrt[4]{15}$

4. Identify the number in each case.

a) 5 is a square root of the number.

b) 6 is the cube root of the number.

c) 7 is a fourth root of the number.

5. For $\sqrt[3]{35}$, does its decimal form terminate, repeat, or neither? Support your answer with an explanation.

4.2

6. Tell whether each number is rational or irrational. Justify your answers.

a) -2 **b)** 17 **c)** $\sqrt{16}$

d) $\sqrt{32}$ **e)** 0.756 **f)** $12.\overline{3}$

g) 0 **h)** $\sqrt[3]{81}$ **i)** π

7. Determine the approximate side length of a square with area 23 cm^2. How could you check your answer?

8. Look at this calculator screen.

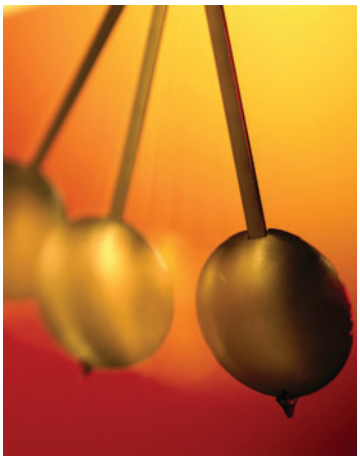
a) Is the number 3.141 592 654 rational or irrational? Explain.

b) Is the number π rational or irrational? Explain your answer.

9. Place each number on a number line, then order the numbers from least to greatest.

$\sqrt[3]{30}$, $\sqrt{20}$, $\sqrt[4]{18}$, $\sqrt[3]{-30}$, $\sqrt{30}$, $\sqrt[4]{10}$

10. The formula $T = 2\pi\sqrt{\frac{L}{9.8}}$ gives the time, T seconds, for one complete swing of a pendulum with length L metres. A clock pendulum is 0.25 m long. What time does the pendulum take to complete one swing? Give the answer to the nearest second.



4.3

11. Write each radical in simplest form.

a) $\sqrt{150}$ **b)** $\sqrt[3]{135}$

c) $\sqrt{112}$ **d)** $\sqrt[4]{162}$

12. Write each mixed radical as an entire radical.

a) $6\sqrt{5}$ **b)** $3\sqrt{14}$

c) $4\sqrt[3]{3}$ **d)** $2\sqrt[4]{2}$

13. Alfalfa cubes are fed to horses to provide protein, minerals, and vitamins.

Two sizes of cubes have volumes 32 cm^3 and 11 cm^3. What is the difference in the edge lengths of the cubes? How can you use radicals to find out?

14. A student simplified $\sqrt{300}$ as shown:

$$\begin{aligned}\sqrt{300} &= \sqrt{3}\cdot\sqrt{100}\\ &= \sqrt{3}\cdot\sqrt{50}\cdot\sqrt{50}\\ &= \sqrt{3}\cdot\sqrt{2}\cdot\sqrt{25}\cdot\sqrt{2}\cdot\sqrt{25}\\ &= 3\cdot 5\cdot\sqrt{2}\cdot 5\\ &= 75\sqrt{2}\end{aligned}$$

Identify the errors the student made, then write a correct solution.

15. Arrange these numbers in order from greatest to least, without using a calculator. Describe your strategy.

$5\sqrt{2}, 4\sqrt{3}, 3\sqrt{6}, 2\sqrt{7}, 6\sqrt{2}$

4.4

16. Show, with examples, why $a^{\frac{1}{n}} = \sqrt[n]{a}$, when n is a natural number and a is a rational number.

17. Express each power as a radical.

a) $12^{\frac{1}{4}}$ **b)** $(-50)^{\frac{5}{3}}$

c) $1.2^{0.5}$ **d)** $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

18. Express each radical as a power.

a) $\sqrt{1.4}$ **b)** $\sqrt[3]{13^2}$

c) $\left(\sqrt[5]{2.5}\right)^4$ **d)** $\left(\sqrt[4]{\frac{2}{5}}\right)^3$

19. Evaluate each power without using a calculator.

a) $16^{0.25}$ **b)** $1.44^{\frac{1}{2}}$

c) $(-8)^{\frac{5}{3}}$ **d)** $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

20. Radioactive isotopes decay. The half-life of an isotope is the time for its mass to decay by $\frac{1}{2}$. For example, polonium-210 has a half-life of 20 weeks. So, a sample of 100 g would decay to 50 g in 20 weeks. The percent, P, of polonium remaining after time t weeks is given by the formula $P = 100(0.5)^{\frac{t}{20}}$. What percent of polonium remains after 30 weeks?

21. Arrange these numbers in order from greatest to least. Describe the strategy you used.

$\sqrt[4]{5}, 5^{\frac{2}{3}}, \sqrt[3]{5}, 5^{\frac{3}{4}}, \left(\sqrt{5}\right)^3$

22. Kleiber's law relates a mammal's metabolic rate while resting, q Calories per day, to its body mass, M kilograms:

$q = 70M^{\frac{3}{4}}$

What is the approximate metabolic rate of each animal?

a) a cow with mass 475 kg

b) a mouse with mass 25 g

4.5

23. **a)** Identify the patterns in this list.

$$\begin{aligned}81 &= 3^4\\ 27 &= 3^3\\ 9 &= 3^2\end{aligned}$$

b) Extend the patterns in part a downward. Write the next 5 rows in the pattern.

c) Explain how this pattern shows that $a^{-n} = \frac{1}{a^n}$ when a is a non-zero rational number and n is a natural number.

24. Evaluate each power without using a calculator.

a) 2^{-2} **b)** $\left(\frac{2}{3}\right)^{-3}$ **c)** $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$

25. Kyle wants to have \$1000 in 3 years. He uses this formula to calculate how much he should invest today in a savings account that pays 3.25% compounded annually: $P = 1000(1.0325)^{-3}$ How much should Kyle invest today?

26. A company designs a container with the shape of a triangular prism to hold 500 mL of juice. The bases of the prism are equilateral triangles with side length s centimetres. The height, h centimetres, of the prism is given by the formula:

$h = 2000(3)^{-\frac{1}{2}}s^{-2}$

What is the height of a container with base side length 8.0 cm? Give your answer to the nearest tenth of a centimetre.

27. When musicians play together, they usually tune their instruments so that the note A above middle C has frequency 440 Hz, called the *concert pitch*. A formula for calculating the frequency, F hertz, of a note n semitones above the concert pitch is:

$F = 440(\sqrt[12]{2})^n$

Middle C is 9 semitones below the concert pitch. What is the frequency of middle C? Give your answer to the nearest hertz.

4.6

28. Simplify. Explain your reasoning.

a) $(3m^4n)^2$ **b)** $\left(\frac{x^2y}{y^{-2}}\right)^{-2}$

c) $(16a^2b^6)^{-\frac{1}{2}}$ **d)** $\left(\frac{r^3s^{-1}}{s^{-2}r^{-2}}\right)^{-\frac{2}{3}}$

29. Simplify. Show your work.

a) $(a^3b)(a^{-1}b^4)$ **b)** $\left(x^{\frac{1}{2}}y\right)\left(x^{\frac{3}{2}}y^{-2}\right)$

c) $\frac{a^3}{a^5} \cdot a^{-3}$ **d)** $\frac{x^2y}{x^{\frac{1}{2}}y^{-2}}$

30. Evaluate.

a) $\left(\frac{3}{2}\right)^{\frac{3}{2}} \cdot \left(\frac{3}{2}\right)^{\frac{1}{2}}$ **b)** $\frac{(-5.5)^{\frac{2}{3}}}{(-5.5)^{-\frac{4}{3}}}$

c) $\left[\left(-\frac{12}{5}\right)^{\frac{1}{3}}\right]^6$ **d)** $\frac{0.16^{\frac{3}{4}}}{0.16^{\frac{1}{4}}}$

31. A sphere has volume 1100 cm^3. Explain how to use exponents or radicals to estimate the radius of the sphere.

32. Identify any errors in each solution, then write a correct solution.

a) $\left(s^{-1}t^{\frac{1}{3}}\right)(s^4t^3) = s^{-1} \cdot s^4 \cdot t^{\frac{1}{3}} \cdot t^3$

$= s^{-4}t$

b) $\left(\frac{4c^{\frac{1}{3}}}{d^3}\right)^{-3} = \frac{-12c^{-1}}{d^0}$

$= -12c^{-1}$

$= \frac{1}{12c}$

THE WORLD OF MATH

Historical Moment: The Golden Ratio

The ratio, $\frac{1+\sqrt{5}}{2}$: 1, is called the *golden ratio*.

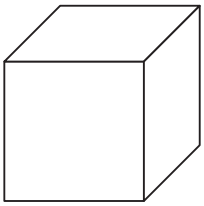
Buildings and pictures with dimensions in this ratio are often considered visually pleasing and "natural." The Greek sculptor Phidias used the golden ratio for the dimensions of his sculptures. His 42-ft. high statue of the Greek god Zeus in the temple in Olympia, created in about 435 B.C.E., was one of the Seven Wonders of the Ancient World. The number $\frac{1+\sqrt{5}}{2}$ is often called "phi" after the first Greek letter in "Phidias."

PRACTICE TEST

For questions 1 and 2, choose the correct answer: A, B, C, or D

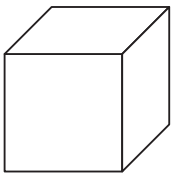
1. The volume V cubic inches of each cube is given. For which cube is the edge length an irrational number?

A.



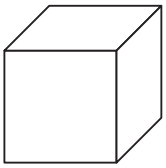
$V = 125$ in.^{3}

B.



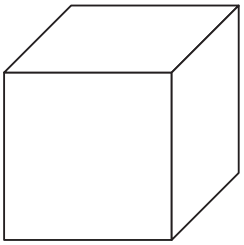
$V = 75$ in.^{3}

C.



$V = 64$ in.^{3}

D.



$V = 216$ in.^{3}

2. Which number is rational?

A. $\sqrt{0.09}$ **B.** $\sqrt{50}$ **C.** $\sqrt[3]{-\frac{64}{121}}$ **D.** π

3. **a)** Which is greater, $\sqrt{70}$ or $5\sqrt{3}$? Justify your answer.
b) Sketch a number line to illustrate the numbers in part a.

4. Evaluate without using a calculator.

a) $\sqrt[4]{\frac{256}{81}}$ **b)** $(-4)^{-2}$ **c)** $0.81^{\frac{3}{2}}$ **d)** $16^{-\frac{1}{2}}$

5. Write $44^{\frac{1}{2}}$ as a radical in simplest form.

6. A student simplified $\frac{x^{-1}y^3}{xy^{-2}}$ as follows:

$$\begin{aligned}\frac{x^{-1}y^3}{xy^{-2}} &= x^{-1+1} \cdot y^{3-2} \\ &= x^0y^1 \\ &= y\end{aligned}$$

Is the student correct? If not, describe any errors and write a correct solution.

7. Simplify each expression. Write your answers using positive exponents.

a) $(p^{-2}q^{-1})^2\left(pq^{\frac{1}{2}}\right)^2$ **b)** $\left(\frac{c^6d^5}{c^3d^4}\right)^{-\frac{1}{3}}$

8. Scientists use the formula $d = 0.099m^{\frac{9}{10}}$ to calculate the volume of water, d litres, that a mammal with mass m kilograms should drink in 1 day. Calculate how much water a 550-kg moose should drink in one day.