

Test : Curve Sketching and Related Rates

1. Consider the function : $f(x) = \frac{x+1}{(x-3)^2}$

given $f'(x) = \frac{-(x+5)}{(x-3)^3}$ and $f''(x) = \frac{2(x+9)}{(x-3)^4}$

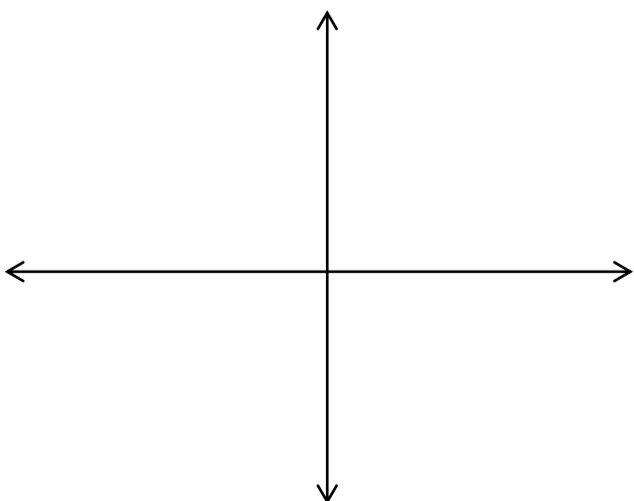
Supply the information requested in the boxes at right and give a careful sketch of f on the axes below.

NAME:

(value = 20)

(John Abbott College: Final Exam 2013)

x-intercept(s)
y-intercept(s)
Vertical asymptote(s)
Horizontal asymptote(s)
Region(s) of increase
Region(s) of decrease
Local maxima
Local minima
Region(s) where concave up
Region(s) where concave down
Point(s) of inflection



2. Coffee is draining through a cone shaped filter at a constant rate of $10 \text{ cm}^3/\text{min}$. The filter has a diameter of 18 cm and a height of 16 cm. Determine the rate at which the coffee in the filter is falling the instant it is at a depth of 10 cm. [6]

3. A balloon is rising vertically above a level, straight road at a constant rate of 1 ft./sec. Just as the balloon reaches a height of 65 feet, a bicycle passes directly below travelling at 17 ft./sec. Determine the rate at which the distance between the bicycle and the balloon is increasing 3 seconds later. [6]

4. Given the function $f(x) = 2x^3 + 6x^2 - 48x + 6 \dots$

(a) Determine the coordinates of all relative extrema and inflection points on the interval $(-\infty, \infty)$ [10]

(b) Determine the **absolute maximum and minimum** values of $f(x)$ on the interval $[0, 3]$. [6]