Pre-Calculus 12B

Unit Test: Exponential and Logarithmic functions **Name:_____**

II. Open Response: All work for each of the following must be shown in the space provided. **[55 Marks]**

1. Given the exponential function: $f(x) = 3(2)^{-5x+5} + 8$

(a) Complete the chart shown below. [16]

	Solution
Growth or Decay →	
Equation of horizontal asymptote →	
Domain →	
Range →	
Horizontal Stretch Factor →	
Vertical Stretch Factor →	
Horizontal Translation →	
Vertical Translation →	
y-Intercept →	
Reflected in *x*-axis? ***Yes or No*** Reflected in *y*-axis? ***Yes or No***	
Mapping rule that maps $y = 3^x$ to this function →	
Coordinates of the new focal point → ((0,1) from $y = 3^x$)	

(b) Determine the coordinates of any point, **other than the focal point**, that would lie on the graph of *f*(*x*). [2]

2. Solve each of the following equations: [8]

(a) $\dfrac{8^{5-2x} \bullet 16^{x+3}}{4^{3x}} = 32^{2x-3}$

(b) $3^{x-3} = \dfrac{6^{1-3x}}{2^x}$

3. Solve for x in each of the following equations: [8]

(a) $\log_6(x+3)+\log_6(x-2)=1$

(b) $\log_7(2x+2)-\log_7(x-1)=\log_7(x+1)$

4. (a) Given that $\log_r x=6$, $\log_r y=-15$, and $\log_r z=5$, evaluate the expression $\log_r\left(\frac{\sqrt[3]{x^2}\,r^5}{\sqrt[5]{yz^4}}\right)$ [4]

(b) Express the following as a **single logarithm in simplest form**: $6\left(\log_2 x^5-\frac{2}{3}\log_2\sqrt[3]{x}\right)+\frac{3}{5}\log_2 x^{15}$ [4]

5. A cup of coffee contains approximately 110 mg of caffeine. When you drink coffee, the caffeine is absorbed into the bloodstream and eventually metabolized by the body. Every three hours the amount of caffeine in the bloodstream is reduced by 48 %.

(a) Write an equation which express the amount of caffeine, C (in mg), remaining in the bloodstream as an exponential function of the elapsed time, t (in hours), since drinking one cup of coffee. [3]

b) Find the amount of caffeine in the bloodstream after 8 hours. [1]

c) How many hours **(accurate to the nearest hundredth)** does it take for the amount of caffeine to be reduced to 45 mg? [4]

5. The Dead Sea Scrolls are a collection of ancient manuscripts discovered in caves along the west bank of the Dead Sea. (The discovery occurred by accident when an Arab herdsman of the Taamireh tribe was searching for a stray goat.) When the linen wrappings on the scrolls were analyzed, it was determined that there was 72.3% of the original Carbon-14 remaining. Given that the half-life of Carbon-14 is known to be 5730 years, estimate the age of the scrolls using the process of Carbon dating. [5]