1. (a) $\frac{2y}{y^2+1}dy = x^2dx$

$\ln(y^2+1) + C = \frac{x^3}{3}$

$\ln(5) + C = 0$

$C = -\ln 5$

$\ln(y^2+1) - \ln 5 = \frac{x^3}{3}$

$\ln\left(\frac{y^2+1}{5}\right) = \frac{x^3}{3}$

$e^{x^3/3} = \frac{y^2+1}{5}$

$y = \sqrt{5e^{x^3/3} - 1}$

b) $2y\,dy = \frac{x}{x^2+1}dx$

$y^2 + C = \frac{1}{2}\ln(x^2+1)$

$4 + C = \frac{1}{2}\ln(1)$

$C = -4$

$y^2 - 4 = \frac{1}{2}\ln(x^2+1)$

$y^2 = \frac{1}{2}\ln(x^2+1) + 4$

$y = \sqrt{\ln\sqrt{x^2+1} + 4}$

c) $y' = 4x^3+1+\frac{4}{x}$

$y'' = 12x^2 - 4x^{-2}$

$\underline{\underline{LS}}$ $\qquad\qquad$ $\underline{\underline{RS}}$

$x\left(12x^2 - \frac{4}{x^2}\right) + 4x^3 + 1 + \frac{4}{x}$ $\Bigg\}$ $16x^3$

$12x^3 - \frac{4}{x} + 4x^3 + 1 + \frac{4}{x}$

$16x^3 + 1$

Not a solution

2. (a) $\int \sin^3x(\cos x)^{1/2}dx$

$\int \sin x(1-\cos^2x)(\cos x)^{1/2}dx$

$\int (\cos x)^{1/2}\sin x - (\cos x)^{5/2}\sin x\,dx$

$= -\frac{2}{3}(\cos x)^{3/2} + \frac{2}{7}(\cos x)^{7/2} + C$

d) $\int \frac{x^3}{1+x^2}dx$

$\Rightarrow x^2+1\,\overline{)\,x^3}$ quotient x

$x^3 + x$

$-x$

$\int\left(x - \frac{1}{2}\frac{x(2)}{x^2+1}\right)dx$

$= \frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) + C$

b) $\int (4x^3+2x)\ln(x^2+1)dx$

$u = \ln(x^2+1)$ $\quad dv = 4x^3+2x$

$du = \frac{2x}{x^2+1}$ $\quad v = x^4+x^2$

$= (x^4+x^2)\ln(x^2+1) - \int \frac{x^4+x^2\,(2x)}{x^2+1}dx$

$= (x^4+x^2)\ln(x^2+1) - \int \frac{x^2\cancel{(x^2+1)}\,2x}{\cancel{x^2+1}}dx$

$= (x^4+x^2)\ln(x^2+1) - \int 2x^3dx$

$= (x^4+x^2)\ln(x^2+1) - \frac{1}{2}x^4 + C$

e) $x^2+8\,\overline{)\,3x^4+29x^2+2x+40}$ quotient $3x^2+5$

$3x^4+24x^2$

$5x^2+2x$

$5x^2+40$[illegible]

[illegible]x [illegible]

$\int(3x^2+5)dx + \int\frac{2x}{x^2+8}dx$

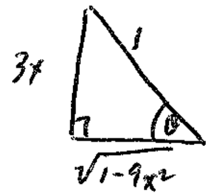
~~$\int(3x^2+5)dx - \frac{38}{2}\int\frac{x(2)}{x^2+8}dx + \int\frac{40}{x^2+8}dx$~~

~~$\int(3x^2+5)dx - \frac{38}{2}\int\frac{2x}{x^2+8} + \frac{40}{8}\int\frac{\left(\frac{1}{\sqrt{8}}\right)dx}{8\left[\left(\frac{1}{8}x\right)^2+1\right]}$~~

~~$= x^3 + 5x - 19\ln(x^2+8) + 10\sqrt{2}\tan^{-1}\left(\frac{1}{2\sqrt{2}}x\right) + C$~~

$= x^3 + 5x + \ln(x^2+8) + C$

c) $\int x^3\sqrt{1-9x^2}dx$



$\cos\theta = \sqrt{1-9x^2}$

$\sin\theta = 3x$

$x = \frac{1}{3}\sin\theta$

$dx = \frac{1}{3}\cos\theta\,d\theta$

$\int\left(\frac{1}{3}\sin\theta\right)^3\cos\theta\left(\frac{1}{3}\cos\theta\right)d\theta$

$\frac{1}{81}\int\sin^3\theta\cos^2\theta\,d\theta$

~~$= \frac{1}{81}\left[\frac{1}{4}\sin^4\theta\right] + C$~~

~~$= \frac{1}{324}\sin^4\theta + C$~~

$\frac{1}{81}\int\sin\theta(1-\cos^2\theta)\cos^2\theta\,d\theta$

$= \frac{1}{81}\int(\cos^2\theta\sin\theta - \cos^4\theta\sin\theta)d\theta$

$= \frac{1}{81}\left(-\frac{\cos^3\theta}{3} + \frac{\cos^5\theta}{5}\right) + C$

$= -\frac{\cos^3\theta}{243} + \frac{\cos^5\theta}{405} + C$

$= \frac{-1}{243}\left(\sqrt{1-9x^2}\right)^3 + \frac{1}{405}(1-9x^2)^{5/2} + C$

F) $\int \frac{7x^2+4x-5}{(x^2+1)(x-3)}dx$

$\frac{Ax+B}{x^2+1} + \frac{C}{x-3} = \frac{7x^2+4x-5}{(x^2+1)(x-3)}$

$(Ax+B)(x-3)+C(x^2+1) = 7x^2+4x-5$

$Ax^2-3Ax+Bx-3B+Cx^2+C = 7x^2+4x-5$

$A+C=7$ $\quad -3A+B=4$ $\quad -3B+C=-5$

$C=7-A$ $\quad -3B+(7-A)=-5$

$-3B-A=-12$

$\begin{cases} (A+3B=12)3 \\ -3A+B=4 \end{cases}$

$8B=32$

$\underline{B=4}$

$-3A+4=4$

$-3A=0$

$\underline{A=0}$

$\underline{C=7}$

$\int \frac{4}{x^2+1}dx + \int \frac{7}{x-3}dx$

$= 4\tan^{-1}x + 7\ln|x-3| + C$

i) $\int \sin^4(x)\,dx$

$\int (\sin^2 x)^2 dx$ $\quad 1-2\sin^2\theta = \cos 2\theta$

~~$\int (1-\cos^2 x)^2 dx$~~ $\quad 2\cos^2\theta - 1 = \sin 2\theta$

$\cos^2\theta = \sin 2\theta + 1$

~~$\int (1-2\cos^2 x + \cos^4 x)dx$~~

$\int \left[\frac{1}{2}(1-\cos 2\theta)\right]^2 d\theta$

$\frac{1}{4}\int (1-2\cos 2\theta + \cos^2 2\theta)\,d\theta$

$\frac{1}{4}\int (1-2\cos 2\theta)\,d\theta + \frac{1}{4}\int$ ~~$1-2\cos 2\theta +$~~ $\frac{1}{2}(1+\cos 4\theta)\,d\theta$

$\int \frac{1}{4}d\theta \quad \frac{1}{2}\int \cos 2\theta\,d\theta + \frac{1}{8}\int d\theta + \frac{1}{8}\int \cos 4\theta\,d\theta$

$\frac{1}{4}\theta - \frac{1}{4}\sin 2\theta + \frac{1}{8}\theta + \frac{1}{32}\sin 4\theta + C$

g) $\int \sec^7 x \tan^3 x\,dx$

$\int (\sec^6 x \tan^2 x)\sec x \tan x\,dx$

$\int \sec^6 x(\sec^2 x - 1)\sec x \tan x\,dx$

$\int \sec^8 x(\sec x \tan x) - \sec^6 x(\sec x \tan x)\,dx$

$= \frac{1}{9}\sec^9 x - \frac{1}{7}\sec^7 x + C$

(h) $\int x^2 e^{4x}dx$

$u = x^2 \quad dv = e^{4x}$

$du = 2x\,dx \quad v = \frac{1}{4}e^{4x}$

$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\int x e^{4x}dx$

$u = x \quad dv = e^{4x}dx$

$du = 1 \quad v = \frac{1}{4}e^{4x}$

$= \frac{1}{4}x^2e^{4x} - \frac{1}{2}\left[\frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x}dx\right]$

$= \frac{1}{4}x^2e^{4x} - \frac{1}{8}xe^{4x} + \frac{1}{32}e^{4x} + C$

3. a) $\lim_{x\to\infty} \frac{2x}{\ln x + x\left(\frac{1}{x}\right)}$

$\lim_{x\to\infty} \frac{2}{\left(\frac{1}{x}\right)}$

$= \frac{2}{0}$

$\boxed{\therefore \text{D.N.E}}$

b) $\lim_{x\to 0} \frac{4x^3e^x + x^4e^x}{\sin x + x}$ $\quad 8x^3e^x$

$\lim_{x\to 0} \frac{12x^2e^x + 4x^3e^x + 4x^3e^x + x^4e^x}{-\cos x + 1}$

$\lim_{x\to 0} \frac{24xe^x + 12x^2e^x + 24x^2e^x + 8x^3e^x + 4x^3e^x + x^4e^x}{\sin x}$

$\lim_{x\to 0} \frac{24xe^x + 36x^2e^x + 12x^3e^x + x^4e^x}{\sin x}$

$\lim_{x\to 0} \frac{24e^x + 72xe^x + 36x^2e^x + 36x^2e^x + 12x^3e^x + 4x^3e^x + x^4e^x}{\cos x}$

$= \frac{24}{1} = 24$

c) $\lim_{x\to\frac{\pi}{2}} \frac{(\sin x)^{\cos x}}{2(\pi - 2x)(-2)}$

$\lim_{x\to\frac{\pi}{2}} \frac{-\csc^2 x}{8}$

$\boxed{= \frac{-1}{8}}$

4. a) $\int_1^{16} \left(t^{1/2} + t^{-1/2}\right) dt$

$= \frac{2}{3}t^{3/2} + 2t^{1/2} \Big|_1^{16}$

$= \left[\frac{2}{3}(4)^3 + 2(4)\right] - \left[\frac{2}{3} + 2\right]$

$= \frac{126}{3} + 8 - 2$

$= \boxed{48}$

b) $= \tan^{-1}(\sin t)\Big|_0^{\pi/2}$

$= \tan^{-1}(1) - \tan^{-1}(0)$

$= \boxed{\frac{\pi}{4}}$

c) $\int_0^{1/2} \frac{1\,dt}{\sqrt{1-t^2}} - \int_0^{1/2} \frac{t^{(-2)}}{\sqrt{1-t^2}}dt$

$= \sin^{-1} t + \frac{1}{2}\left[\cancel{2}(1-t^2)^{1/2}\right]\Big|_0^{1/2}$

$= \left[\sin^{-1}\frac{1}{2} + \left(1 - \frac{1}{4}\right)^{1/4}\right] - \left[\sin^{-1} 0 + (1-0)^{1/2}\right]$

$= \frac{\pi}{6} + \frac{\sqrt{3}}{2} - 0 - 1$

$= \boxed{\frac{\pi + 3\sqrt{3} - 6}{6}}$

d) $\int_0^{\frac{\pi}{4}} \left(1 + \tan^2 x\right)\sec^2 x$

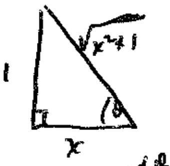
$\int_0^{\pi/4} \sec^2 x + \sec^2 x \tan^2 x \, dx$

$= \tan x + \frac{\tan^3 x}{3}\Big|_0^{\pi/4}$

$= \left(1 + \frac{1}{3}\right) - (0)$

$= \boxed{\frac{4}{3}}$

e) $\int_1^{\sqrt{3}} \frac{dx}{x^2\sqrt{x^2+1}}$



$\cot\theta = x$
$-\csc^2\theta\, d\theta = dx$
$\csc\theta = \sqrt{x^2+1}$

$-\int_1^{\sqrt{3}} \frac{\csc^2\theta\, d\theta}{\cot^2\theta \csc\theta}$

$-\int_1^{\sqrt{3}} \frac{\csc\theta}{\cot^2\theta} d\theta$

$-\int_1^{\sqrt{3}} \left(\frac{1}{\sin\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$

$-\int_1^{\sqrt{3}} \sin\theta\, (\cos\theta)^{-2}\, d\theta$

$\mp(\cos\theta)^{-1}\Big|_1^{\sqrt{3}} = \frac{-1}{\cos\theta}\Big|_1^{\sqrt{3}} = \frac{-\sqrt{x^2+1}}{x}\Big|_1^{\sqrt{3}} = -\frac{2}{\sqrt{3}} + \sqrt{2}$

$= \frac{-2 + \sqrt{6}}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3} + 3\sqrt{2}}{3}}$

5.

x	y
1	0.71
2	1.57
3	3.69
4	8.98
5	22.29
6	56.10

$= \frac{1}{2}(1)\left[(0.71+1.57)+(1.57+3.69)+(3.69+8.98)+(8.98+22.29)+(22.29+56.10)\right]$

$= \frac{1}{2}(129.87)$

$=$ 64.93[illegible]1

$x^2-1=e^x$

6. $\frac{1}{e^2-e}\int_e^{e^2}\frac{1}{x\ln x}dx$

$u=\ln x \Rightarrow \cancel{x}=e^2$

$du=\frac{1}{x}dx \quad u=\ln e^2=2$

$x=e$

$u=\ln e$

$=1$

$\frac{1}{e^2-e}\int_1^2\frac{du}{u}$

$\frac{1}{e^2-e}\ln u\Big|_1^2$

$=\frac{(\ln 2-\ln 1)}{e^2-e}$

$=\boxed{\frac{\ln 2}{e^2-e}}$

7. $\left.\begin{matrix}f(-6)=8\\ f(8)=6\end{matrix}\right\}\begin{matrix}(-6,8)\\(8,6)\end{matrix}$ $m=\frac{2}{-14}=\frac{-1}{7}$

$f'(x)=\frac{1}{\cancel{2}}(100-x^2)^{-\frac{1}{2}}(-\cancel{2}x)$

$\frac{+x}{\sqrt{100-x^2}}=\frac{+1}{7}$

$7x=\sqrt{100-x^2}$

$49x^2=100-x^2$

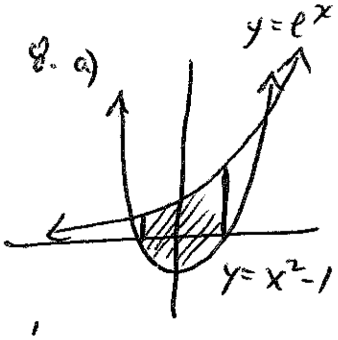
$50x^2=100$

$x^2=2$

$x=\pm\sqrt{2}$

$\boxed{\therefore c=\pm\sqrt{2}}$

8. a)



$\int_{-1}^{1}(e^x-x^2+1)dx$

$=e^x-\frac{x^3}{3}+x\Big|_{-1}^{1}$

$=\left(e-\frac{1}{3}+1\right)-\left(\frac{1}{e}+\frac{1}{3}-1\right)$

$=\frac{4}{3}+e-\frac{1}{e}=\frac{4e+3e^2-3}{3e}$

$=\underline{3.684}$

b)

~~$y=x^2-1$~~ ~~$y^2=x-1$~~

~~$x^2-1=\sqrt{x-1}$~~

~~$x^4-2x^2+1=x-1$~~

~~$x^4-2x^2-x+2=0$~~

~~$x=1$~~

~~$1\,|\,1\;0\;-2\;-1\;2$~~

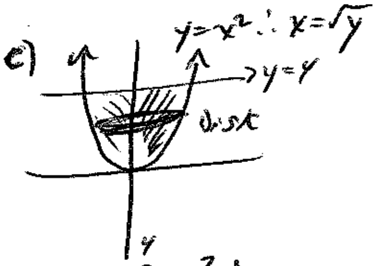
~~$1\;1\;-1\;-2$~~

~~$1\;1\;-2\;0$~~

~~$(x-1)(x^3+x^2-x-2)$~~

~~$x=2$~~

c)



$V=\pi\int_0^4(\sqrt{y})^2dy$

$V=\pi\int_0^4 y\,dy$

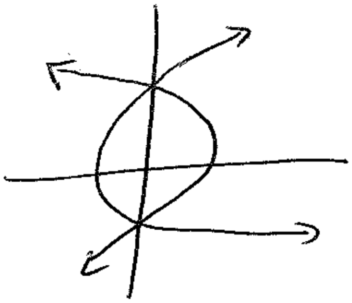
$=\pi\left(\frac{y^2}{2}\right)\Big|_0^4$

$=8\pi-0$

$=\boxed{8\pi\,u^3}$

b) $x=y^2-1$

$x=1-y^2$



$y^2-1=1-y^2$

$2y^2-2=0$

$2(y^2-1)=0$

$y=\pm1$

$\int_{-1}^{1}(1-y^2)-(y^2-1)dy$

$\int_{-1}^{1}2-2y^2dy$

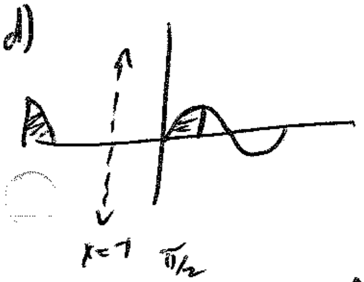
$=2y-\frac{2}{3}y^3\Big|_{-1}^{1}$

$=\left(2-\frac{2}{3}\right)-\left(-2+\frac{2}{3}\right)$

$=4-\frac{4}{3}$

$=\boxed{\frac{8}{3}u^2}$

d)



$V = 2\pi \int_0 (x+1)\sin x \, dx$

$u = x+1 \qquad dv = \sin x$

$du = dx \qquad v = -\cos x$

$= 2\pi \left[-(x+1)\cos x + \int \underset{\sin x}{\cos x}\, dx\right]$

$= 2\pi\left[-\left(\frac{\pi}{2}+1\right)\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right] - \left[-(0+1)\cos 0 + \sin 0\right]$

$= 2\pi\left[(0+1) - (-1+0)\right]$

$= \underline{4\pi\, u^3}$

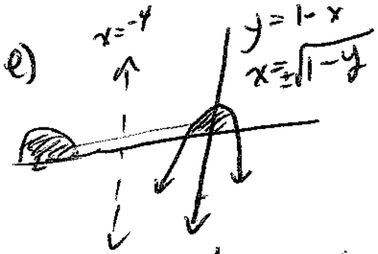
$h'(x) = \frac{(3x)^2 - 1}{(3x)^2 + 3}(3)$

$h'(1) = \frac{9-1}{9+3}(3)$

$= \frac{8}{12}(3)$

$= \boxed{2}$

e)



(a) $2\pi \int_{-1}^{1} (x+4)(1-x^2)\, dx$

$2\pi \int_{-1}^{1} (x - x^3 + 4 - 4x^2)\, dx$

$2\pi \cancel{\int}_{-3}^{1} \left(\frac{x^2}{2} - \frac{x^4}{4} + 4x - \frac{4}{3}x^3\right)\Big|_{1}^{1}$

$2\pi\left[\left(\frac{1}{2} - \frac{1}{4} + 4 - \frac{4}{3}\right) - \left(\frac{1}{2} - \frac{1}{4} - 4 + \frac{4}{3}\right)\right]$

$2\pi\left(\frac{8}{1} - \frac{8}{3}\right) = 2\pi\left(\frac{16}{3}\right) = \boxed{\frac{32\pi}{3}}$

b) $\pi \int_0^1 (\sqrt{1-y} + 4)^2 - (4 - \sqrt{1-y})^2\, dy$

$\pi \int_0^1 \left[(1-y+8\sqrt{1-y}+16) - (16 - 8\sqrt{1-y} + 1-y)\right]$

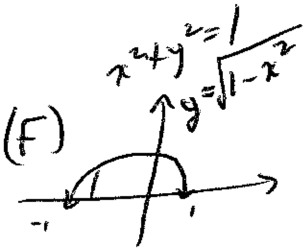
$16\pi \int_0 \sqrt{1-y}\, dy$

$-16\pi \left(\frac{2}{3}(1-y)^{3/2}\right)\Big|_0^1$

$= -\frac{32\pi}{3}(0 - 1)$

$= \boxed{\frac{32\pi}{3}}$

(F)



$\int_{-1}^{1} \left(\sqrt{1-x^2}\right)^2 dx$

$\int_{-1}^{1} (1-x^2)\, dx$

$= x - \frac{x^3}{3}\Big|_{-1}^{1}$

$= \left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)$

$= \frac{2}{1} - \frac{2}{3} = \boxed{\frac{4}{3}} u^3$

10. a) $m = m_0 e^{Kt}$

$4 = 16 e^{12K}$

$\frac{1}{4} = e^{12K}$

$\ln\frac{1}{4} = 12K$

$K = \frac{\ln\frac{1}{4}}{12}$

$m = 16 e^{\frac{\ln\frac{1}{4}}{12}t}$

$2 = 16 e^{\frac{\ln 1/4}{12}t}$

$\frac{1}{8} = e^{\frac{\ln 1/4}{12}t}$

$\ln\frac{1}{8} = \frac{\ln\frac{1}{4}}{12}t$

$\frac{12\ln\frac{1}{8}}{\ln\frac{1}{4}} = t$

$t = 18$

∴ 4g → 2g = 6 years

b) $T = T_s + (T_0 - T_s)e^{Kt}$

$T = 27 + (3-27)e^{5K}$

$\frac{-20}{-24} = e^{5K}$

$\frac{5}{6} = e^{5K}$

$\ln\frac{5}{6} = 5K$

$K = \frac{\ln\frac{5}{6}}{5}$

$T(15) = 27 - 24e^{\frac{\ln(\frac{5}{6})}{5}(15)}$

$\boxed{= 13.11°C}$