Foundations of Mathematics 11 Name: Key February 2017
Unit Test - System of Linear Inequalities

Multiple Choice [10 Marks]
Circle the letter corresponding to the correct solution.

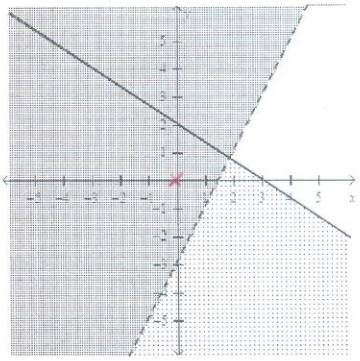
1. For which inequality is (–50, –50) a possible solution?
A) $y - 2x \geq 10$ B) $y \leq -9 + 2x$ C) $y > 9$ D) $y < x - 2$

A

2. What is the boundary line for the linear inequality $4x + 2y < 18$?
A) $y = -4x + 36$ B) $y = -2x + 18$ C) $x = -2y + 18$ D) $y = -2x + 9$

D

3. What system of linear inequalities is shown here?



B

A) $2x + 3y < 6$
$y > 2x - 3$

B) $2x + 3y \leq 6$
$y > 2x - 3$

C) $2x + 3y \leq 6$
$y \geq 2x - 3$

D) $2x + 3y < 6$
$y \geq 2x - 3$

4. Which location best describes where would you find the optimal solutions to an objective function?
A) within the feasible region B) along a boundary line
C) outside the feasible region D) at or near the points of intersection

D

5. Describe the boundary lines for the following system of linear inequalities: $\{y - 3x < 12, x + y \geq 0, x \in R, y \in R\}$
A) Solid line along $y = 3x + 12$; dashed line along $y = -x$ B) Solid line along $y = 3x + 12$; solid line along $y = -x$
C) Dashed line along $y = 3x + 12$; solid line along $y = -x$ D) Dashed line along $y = 3x + 12$; dashed line along $y = -x$

C

6. A football stadium has 60 000 seats.
- 70% of the seats are in the lower deck.
- 30% of the seats are in the upper deck.
- At least 40 000 tickets are sold per game.
- A lower deck ticket costs \$100, and an upper deck ticket costs \$60.

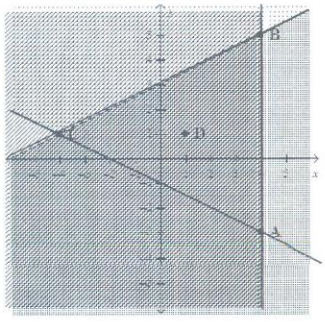
Let x represent the number of lower deck tickets. Let y represent the number of upper deck tickets.
How would you write the objective function for revenue, R?
A) $R = 100x + 60y$ B) $R = 30x + 70y$ C) $R = 60x + 100y$ D) $R = 70x + 30y$

A

7. Which point in the model below would result in the **maximum** value of the objective function $H = x - y$?

A) (4, -3)
B) (4, 5)
C) (-4, 1)
D) (1, 1)

A



Use the following to answer questions 8, 9 & 10

Noah volunteers to fold origami swans and frogs for a display in Mr. Hallihan's Math classroom.
- He has 8 squares of white paper for the swans and 12 squares of green paper for the frogs. $s \leq 8$ $f \leq 12$
- It takes her 4 min to fold an origami swan and 3 min to fold an origami frog.
- There must be at least two frogs for every swan. $f \geq 2s$

Let f represent the number of frogs. Let s represent the number of swans.

8. What are the restrictions on f and s?

B A) $f \in R, s \in R$ (B) $f \in W, s \in W$ C) $f \in I, s \in I$ D) No restrictions

9. Which of the following is a constraint for this situation?

D A) $2f \leq s$ B) $2f \geq s$ C) $f \leq 2s$ (D) $f \geq 2s$

10. Which of the following is a constraint for this situation?

C A) $f \leq 8$ B) $f \geq 8$ (C) $s \leq 8$ D) $s \geq 8$

Open Response [40 Marks]

Show ALL your work in the space provided. Be sure to scale and label your graphs when necessary!

ONLY SHADE/STIPPLE(write the word 'stipple' if the area is too big) THE <u>FEASIBLE REGION</u> IN YOUR GRAPHS.

/10

1. a) Graph the following system of linear inequalities: $\{(x, y) | 4x - 8 \leq 0, x \in I, y \in I\}$ [8]
$\{(x, y) | 6x + 3y - 18 > 0, x \in I, y \in I\}$

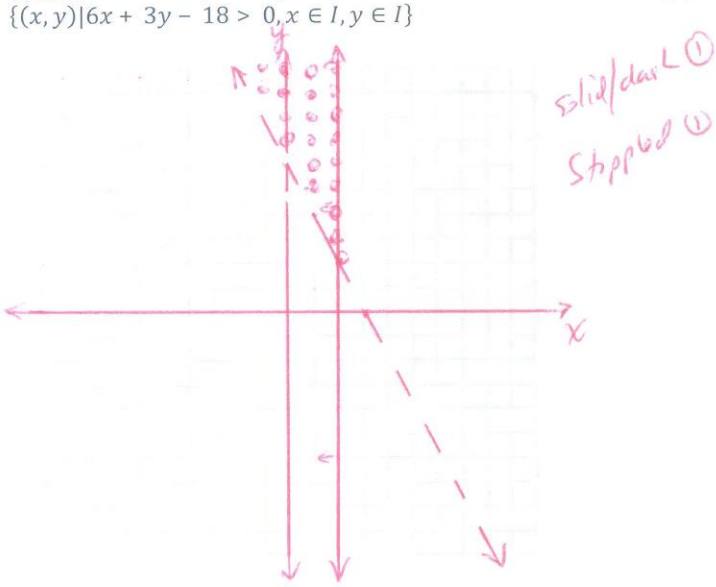
$4x - 8 = 0$ $6x + 3y - 18 = 0$

$\frac{4x}{4} = \frac{8}{4}$ $\frac{3y}{3} = \frac{-6x}{3} + \frac{18}{3}$

$x = 2$ (1) $y = -2x + 6$ (1)

$LS \leq RS$	
$4(0) - 8$	0
-8	yes (1)

$LS > RS$	
$6(0) + 3(0) - 18$	0
-18	NO (1)



solid/dash (1)

stippled (1)

/10

b) Answer each of the following... [2]

i) State a possible solution → (1, 6) (1)

* answers will vary

ii) Is the intersection point a solution? (circle): YES (NO) (1)

/2

2. The following algebraic model represents an optimization problem…

Restrictions: $x \in R; y \in R$

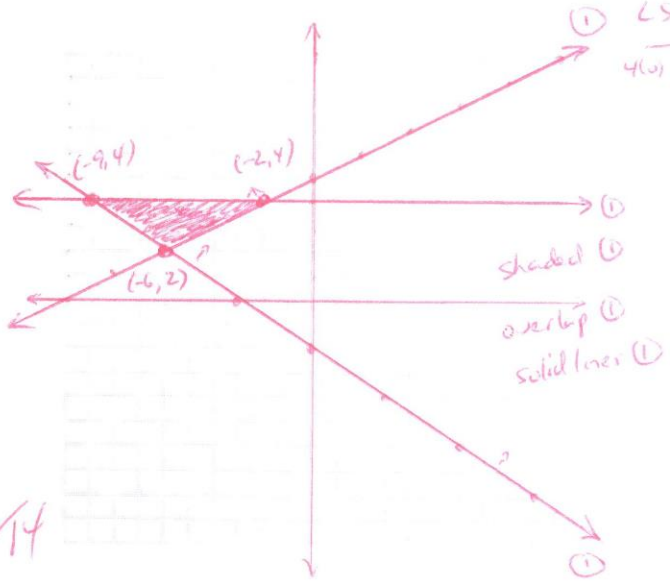
Constraints: $4x + 6y \geq -12$; $2y \geq x + 10$; $y \leq 4$

Objective Function: $M = 2x - 3y$

$\frac{6y}{6} = \frac{-4x}{6} - \frac{12}{6}$ $\quad y = -\frac{2}{3}x - 2$

$\frac{2y}{2} = \frac{x}{2} + \frac{10}{2}$ $\quad y = \frac{1}{2}x + 5$ $\quad y = 4$ ①

a) Create a graphical model to represent the problem. [9]



LS	≥	RS
4(0)+6(0)		-12
0		yes ①

LS	≥	RS
2(0)		0+10
0		10
		NO ①

	$M = 2x - 3y$
(-9,4)	$2(-9) - 3(4) = -30$
(-2,4)	$2(-2) - 3(4) = -16$
(-6,2)	$2(-6) - 3(2) = -18$

shaded ①
overlap ①
solid lines ①

14 ①

b) What are **ALL** the vertices of the feasible region? [3]

(-9, 4) ① ; (-2, 4) ① ; (-6, 2) ①

c) Which point(s) would result in the **maximum** value of the objective function? What is the value? [2]

Point → (-2, 4) ① **Max Value →** -16 ①

3. Anita Summoola has two summer jobs…one at Pita Pit and the other at Robin's Coffee.
- She works no more than a total of 50 h a week. Both jobs allow her to have flexible hours but in whole hours only.
- At the Pita Pit, Anita works no fewer than 25 hours and earns $10.50/hr.
- At the Robin's Coffee, Anita works at most 20 hours and earns $12.25/hr.

a) Define the variables and state any restrictions. [2]

$x \rightarrow$ # of hours at Pita Pit $\quad x \in w$

$y \rightarrow$ # of hours at Robin's $\quad y \in w$

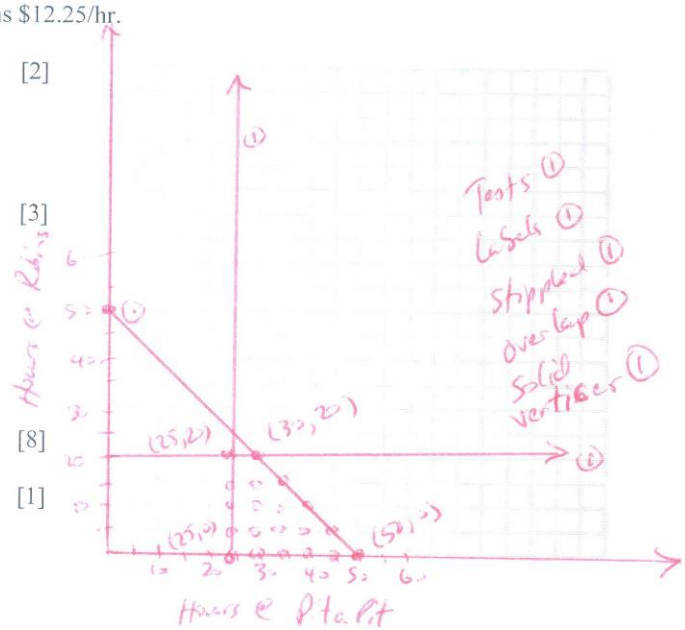
b) List the constraints and any other inequalities. [3]

$x + y \leq 50$ ① $\quad x \geq 25$ ① $\quad y \leq 20$ ①

x int (50, 0)

y int (0, 50)

c) Create a graphical model to represent the problem. [8]



Tests ①
Labels ①
Stippled ①
Overlap ①
Solid vertices ①

d) State the objective function. [1]

16 $E = 10.50x + 12.25y$ ①

e) What combination of numbers of hours will allow Anita to maximize her earnings and what will be her earnings? [2]

$10.50(30) + 12.25(20)$ **(Show your work to justify your solution)**

$= \$560$

COMBINATION OF HOURS → 30 hrs @ Pita Pit, 20 hrs @ Robin's **MAX EARNINGS = $** 560

/30