APRIL 9, 2019

UNIT 7: SIMILARITY AND TRANSFORMATIONS

7.4: SIMILAR TRIANGLES

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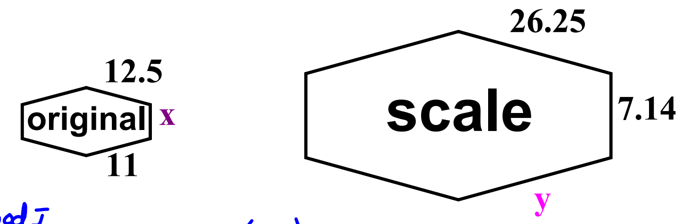
MATH 9

WHAT'S THE POINT OF TODAY'S LESSON?

We will begin working on the Math 9 Specific Curriculum Outcome (SCO) "Shape and Space 3" OR "SS3" which states:

"Demonstrate an understanding of similarity of polygons."

WARM UP: Find the lengths of the missing sides ("x" and "y") in the proportional diagrams below. **Show all work.**



Method I

$S.F. = \frac{S}{O} = \frac{26.25}{12.5} = 2.1$

$y = O(S.F.) = 11(2.1) = 23.1$

$x = \frac{S}{S.F.} = \frac{7.14}{2.1} = 3.4$

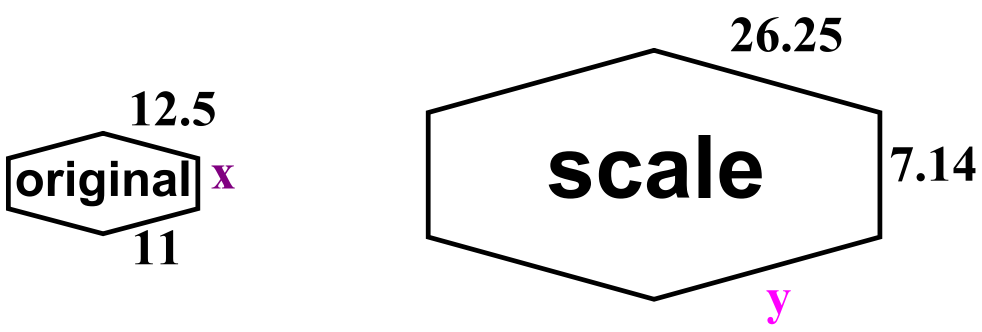
Method II

$\frac{x}{7.14} = \frac{12.5}{26.25}$

$x = \frac{12.5(7.14)}{26.25} = 3.4$

$\frac{y}{11} = \frac{26.25}{12.5}$

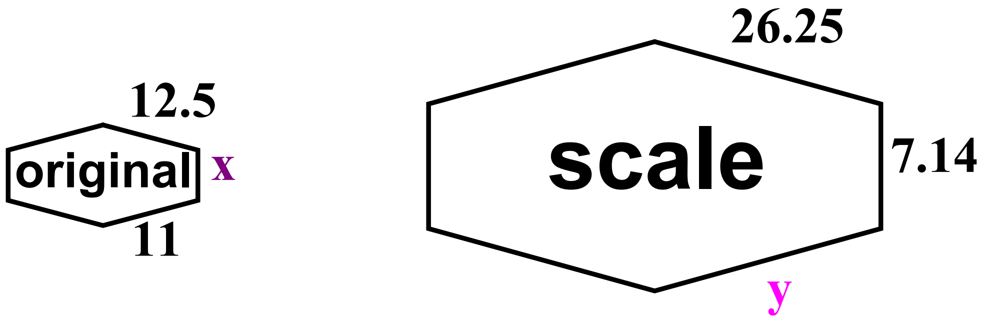
$y = \frac{26.25(11)}{12.5} = 23.1$



A SOLUTION:

Scale Factor = scale/original
= 26.25/12.5
= 2.1

x = 7.14 / 2.1 **y = 11 x 2.1**
x = 3.4 **y = 23.1**



ANOTHER SOLUTION:

$$\frac{26.25}{12.5} = \frac{7.14}{x} = \frac{y}{11}$$

$$\frac{26.25}{12.5} = \frac{7.14}{x}$$

$$\frac{26.25x}{26.25} = \frac{89.25}{26.25}$$

$$x = 3.4$$

$$\frac{26.25}{12.5} = \frac{y}{11}$$

$$\frac{288.75}{12.5} = \frac{12.5y}{12.5}$$

$$23.1 = y$$

HOMEWORK QUESTIONS?

(pages 329 / 330 / 331, #4 to #6, #8 to #11 & #20)

PLEASE TURN TO PAGE 316 IN *MMS9*.

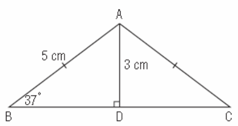
This is a review of several properties you should already know about triangles.

Start Where You Are

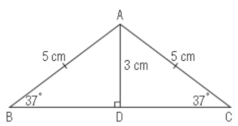
What Should I Recall?

Suppose I have to solve this problem:
Determine the unknown measures of the angles and sides in ΔABC.
The given measures are rounded to the nearest whole number.

I think of what I already know about triangles.
I see that AB and AC have the same hatch marks; this means the sides are equal.
AC = AB
So, AC = 5 cm

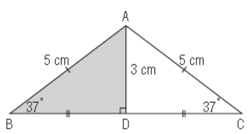


I know that a triangle with 2 equal sides is an isosceles triangle.
So, ΔABC is isosceles.
An isosceles triangle has 2 equal angles that are formed where the equal sides intersect the third side.
I use 3 letters to describe an angle.



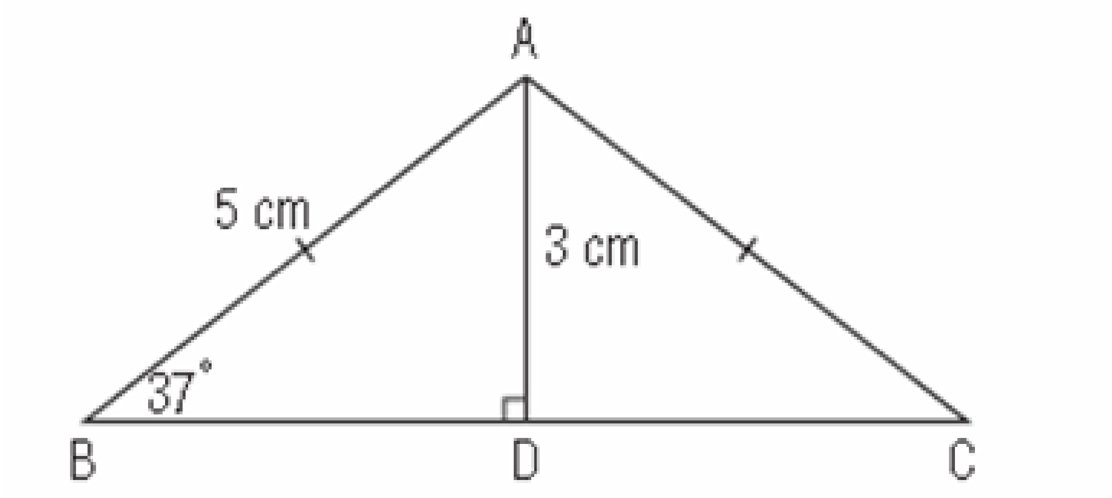
So, $\angle ACD = \angle ABD$
$= 37°$

Since ΔABC is isosceles, the height AD is the perpendicular bisector of the base BC.
So, BD = DC and ∠ADB = 90°
I can use the Pythagorean Theorem in ΔABD to calculate the length of BD.



$$\begin{aligned} AD^2 + BD^2 &= AB^2 \\ 3^2 + BD^2 &= 5^2 \\ 9 + BD^2 &= 25 \\ 9 - 9 + BD^2 &= 25 - 9 \\ BD^2 &= 16 \\ BD &= \sqrt{16} \\ BD &= 4 \end{aligned}$$

316 UNIT 7: Similarity and Transformations

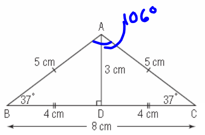


BD = 4 cm
So, BC = 2 × 4 cm
= 8 cm

SATT

I know that the sum of the angles in a triangle is 180°.
So, I can calculate the measure of ∠BAC.

$\angle BAC + \angle ACD + \angle ABD = 180°$
$\angle BAC + 37° + 37° = 180°$
$\angle BAC + 74° = 180°$
$\angle BAC + 74° - 74° = 180° - 74°$
$\angle BAC = 106°$



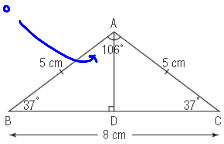
My friend Janelle showed me a different way to calculate.
She recalled that the line AD is a line of symmetry for an isosceles triangle.
So, ΔABD is congruent to ΔACD.

This means that ∠BAD = ∠CAD
Janelle calculated the measure of ∠BAD in ΔABD.

= 180-90-37
= 53°

$\angle BAD + 37° + 90° = 180°$
$\angle BAD + 127° = 180°$
$\angle BAD + 127° - 127° = 180° - 127°$
$\angle BAD = 53°$

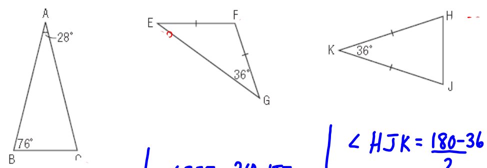
Then, $\angle BAC = 2 \times 53°$
$= 106°$



Check

1. Calculate the measure of each angle.

a) ∠ACB **b)** ∠GEF and ∠GFE **c)** ∠HJK and ∠KHJ



∠ACB = 180-76-28
= 76° SATT

∠GEF = 36° ITT
∠GFE = 180-36-36
= 108° SATT

∠HJK = $\frac{180-36}{2}$
= 72° SATT ITT
∠KHJ = 72° ITT

SIMILAR TRIANGLES

TO IDENTIFY SIMILAR TRIANGLES:

* the measures of the 3 pairs of corresponding angles must be EQUAL

OR

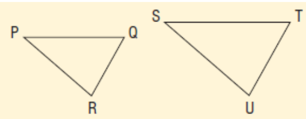
* the ratios of the lengths of the 3 pairs of corresponding sides must be EQUAL; in other words, corresponding sides are proportional

MMS9, Page 344:

Properties of Similar Triangles

To identify that ΔPQR and ΔSTU are similar, we only need to know that:

- $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$; *or*
- $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$



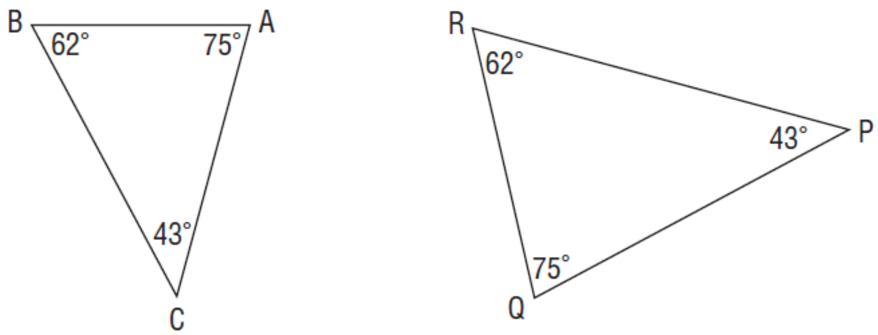
similar

$\Delta PQR \sim \Delta STU$

$\angle P = \angle S$
$\angle Q = \angle T$
$\angle R = \angle U$

$\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$ or $\frac{ST}{PQ} = \frac{TU}{QR} = \frac{SU}{PR}$

ARE THESE TWO TRIANGLES SIMILAR?



$\angle A = \angle Q$ given
$\angle B = \angle R$ given
$\angle C = \angle P$ given

$\therefore \Delta ABC \sim \Delta QRP$ by AAA

So $\frac{AB}{QR} = \frac{AC}{QP} = \frac{BC}{RP}$

EXAMPLE -*MMS9*, PAGE 344:

These triangles are similar because:

∠A = ∠Q = 75°

∠B = ∠R = 62°

∠C = ∠P = 43°

When we name two similar triangles, we order the letters to match the corresponding angles.

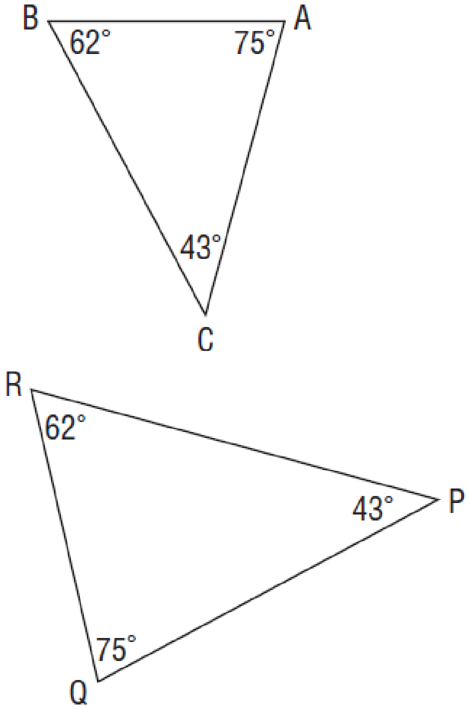
We write: ΔABC ~ ΔQRP

Then we can identify corresponding sides:

AB corresponds to QR.

BC corresponds to RP.

AC corresponds to QP.



EXAMPLE - How you show PROOF OF SIMILARITY (AAA) in your work:

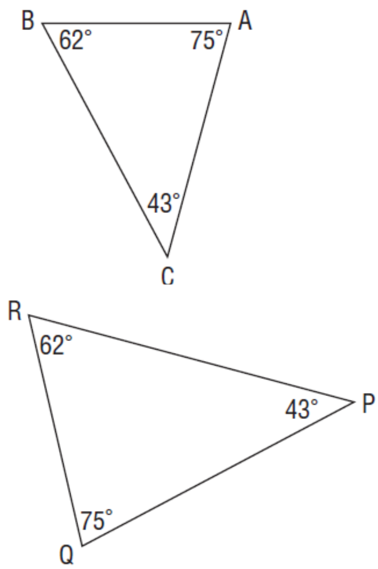
(NOTE: "AAA" = angle; angle; angle)

∠A = ∠Q **(GIVEN)**

∠B = ∠R **(GIVEN)**

∠C = ∠P **(GIVEN)**

∴ △ ABC∼△ QRP (AAA)



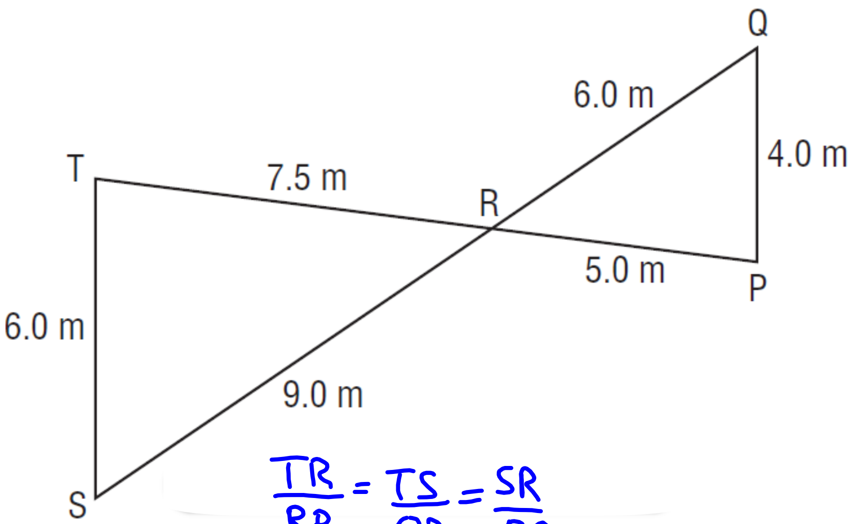
$\Delta ABC \sim \Delta QRP$

SYMBOL FOR "SIMILAR TO"

THIS IS CALLED A

"SIMILARITY STATEMENT"

ARE THESE TWO TRIANGLES SIMILAR?



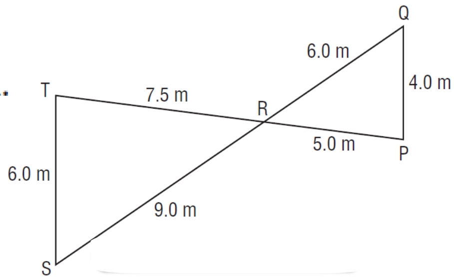
$$\frac{TR}{RP} = \frac{TS}{QP} = \frac{SR}{RQ}$$

$$\frac{7.5}{5.0} \quad \Big| \quad \frac{6.0}{4.0} \quad \Big| \quad \frac{9.0}{6.0}$$

$$1.5 \quad 1.5 \quad 1.5$$

$\therefore \Delta STR \sim \Delta QPR$ by ratio

EXAMPLE -*MMS9*, PAGE 345:



Find out if the corresponding sides are proportional.

$$\frac{ST}{PQ} = \frac{6.0}{4.0} = 1.5$$

$$\frac{TR}{PR} = \frac{7.5}{5.0} = 1.5$$

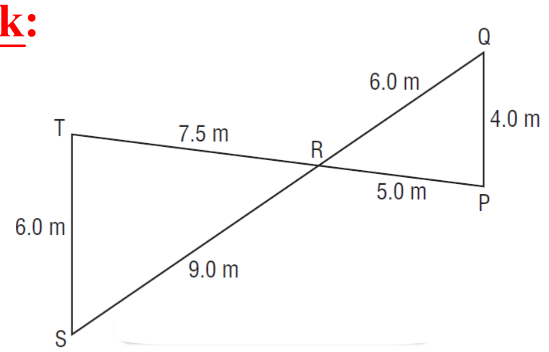
$$\frac{RS}{QR} = \frac{9.0}{6.0} = 1.5$$

Since the corresponding sides are proportional, the triangles are similar.
P and T are the vertices where the two shorter sides in each triangle meet, so ∠P corresponds to ∠T.
Similarly, ∠Q corresponds to ∠S and ∠TRS corresponds to ∠QRP.
So, ΔPQR ~ ΔTSR

EXAMPLE -*MMS9*, PAGE 345 (continued):

In *Example 1*, we can say that ΔTSR is an enlargement of ΔPQR with a scale factor of 1.5.
Or, since $1.5 = \frac{3}{2}$, we can also say that ΔPQR is a reduction of ΔTSR with a scale factor of $\frac{2}{3}$.
We can use the properties of similar triangles to solve problems that involve scale diagrams.
These problems involve lengths that cannot be measured directly.

EXAMPLE - How you show PROOF OF SIMILARITY (RATIOS) in your work:



$$\frac{PQ}{TS} = \frac{QR}{SR} = \frac{PR}{TR} = \frac{2}{3}$$

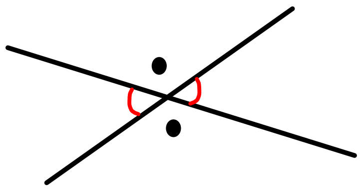
OR

$$\frac{RS}{RQ} = \frac{ST}{QP} = \frac{RT}{RP} = \frac{3}{2} = 1.5$$

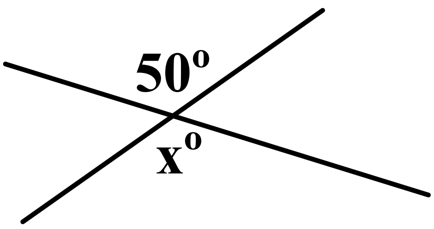
$\therefore \triangle PQR \sim \triangle TSR$ (RATIOS)

There are two angle theorems that you will need for your similar triangles proofs:

1. **OPPOSITE ANGLES THEOREM (OAT):** opposite angles are EQUAL



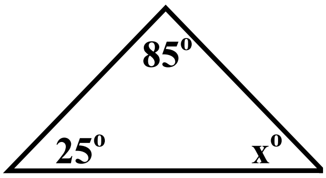
Ex.:



$x = 50°$

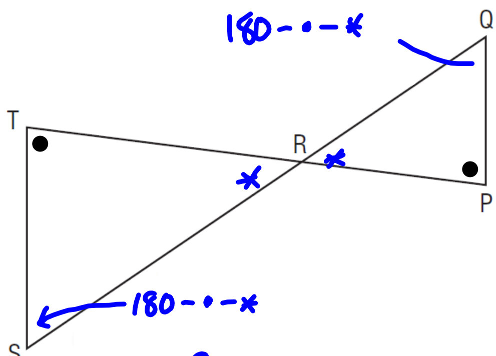
2. SUM OF THE ANGLES IN A TRIANGLE THEOREM (SATT): the sum of the angles in a triangle is 180° .

Ex.: Calculate the unknown angle measure.



x = 180 - 85 - 25
= 180 - 110
= 70°

EXAMPLE: PROVE that the triangles in the diagram below are SIMILAR.



<T = <P (given)
<TRS = <QRP (OAT)
<S = <Q (SATT)

∴ △RST ~ △RQP (AAA)

∠T = ∠P given
∠TRS = ∠QRP OAT
∠S = ∠Q SATT

∴ ΔTSR ~ ΔPQR (AAA)