SEPTEMBER 17, 2019

UNIT 1: ROOTS AND POWERS

SECTION 4.5:
NEGATIVE EXPONENTS
AND RECIPROCALS

K. Sears
NUMBERS, RELATIONS AND FUNCTIONS 10

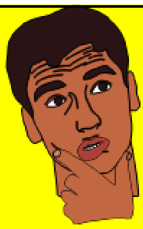
#2

WHAT'S THE POINT OF TODAY'S LESSON?

We will continue working on the NRF 10 Specific Curriculum Outcome (SCO) "Algebra and Numbers 3" OR "AN3" which states:

"Demonstrate an understanding of powers with integral and rational exponents."

#3



What does THAT mean???

SCO AN3 means that we will:

* apply the 6 exponent laws you learned in grade 9:
  $a^0 = 1$
  $(a^m)(a^n) = a^{m+n}$
  $a^m \div a^n = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^m = a^m b^m$
  $(a \div b)^n = a^n \div b^n$
* use patterns to explain $a^{-n} = \frac{1}{a^n}$ and $a^{\frac{1}{n}} = \sqrt[n]{a}$
* apply all exponent laws to evaluate a variety of expressions
* express powers with rational exponents as radicals and vice versa
* identify and correct errors in work that involves powers

WARM-UP:

Write the power below as a radical then evaluate.

$$\left(\frac{64}{125}\right)^{\frac{2}{3}} = \frac{64^{\frac{2}{3}}}{125^{\frac{2}{3}}}$$

$$= \frac{16}{25}$$

WHITE BOARD WARM-UP (Day 2):

First, write the power below with a positive exponent. At this point, write the power as a radical then evaluate.

$$\left(\frac{64}{125}\right)^{\frac{-2}{3}} = \left(\frac{125}{64}\right)^{\frac{2}{3}}$$

$$= \frac{25}{16}$$

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HOMEWORK QUESTIONS???

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#13. a) $\boxed{4}^{\frac{1}{2}} = 2$ c) $\boxed{100}^{\frac{1}{2}} = 10$ e) $\boxed{25}^{\frac{1}{2}} = 5$

b) $\boxed{16}^{\frac{1}{2}} = 4$ d) $\boxed{9}^{\frac{1}{2}} = 3$

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EXPONENT LAWS (separate sheet):

1. **Zero Exponent Law:** $a^0 = 1$
2. **Product of Powers:** $(a^m)(a^n) = a^{m+n}$
3. **Quotient of Powers:** $a^m \div a^n = a^{m-n}$
4. **Power of a Power:** $(a^m)^n = a^{mn}$
5. **Power of a Product:** $(ab)^m = a^m b^m$
6. **Power of a Quotient:** $(a \div b)^n = a^n \div b^n$

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7. MULTIPLICATION PROPERTY OF RADICALS:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

EX.: $\sqrt{24}$ (Factors: 1, 2, 3, 4, 6, 8, 12, 24)

$$\begin{aligned} &\sqrt{24} \\ &= \sqrt{4 \cdot 6} \\ &= \sqrt{4} \cdot \sqrt{6} \\ &= 2 \cdot \sqrt{6} \\ &= 2\sqrt{6} \quad \text{(MIXED RADICAL)} \end{aligned}$$

EX.: $\sqrt[3]{24}$ (ENTIRE RADICAL)

$$\begin{aligned} &\sqrt[3]{24} \\ &= \sqrt[3]{8 \cdot 3} \\ &= \sqrt[3]{8} \cdot \sqrt[3]{3} \\ &= 2 \quad \sqrt[3]{3} \\ &= 2\sqrt[3]{3} \end{aligned}$$

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8. POWERS WITH RATIONAL EXPONENTS WITH A NUMERATOR OF 1:

$$x^{\frac{1}{n}} = \sqrt[n]{x}$$

EX.:

$$8^{\frac{1}{3}}$$
$$= \sqrt[3]{8}$$
$$= 2$$

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9. POWERS WITH RATIONAL EXPONENTS:

EXPONENT

INDEX

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

AND

EXPONENT

INDEX

$$x^{\frac{m}{n}} = \left(x^m\right)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

EX.: Evaluate $16^{\frac{3}{2}}$.

$16^{\frac{3}{2}}$ 3 (EXPONENT), 2 (INDEX)

$= \left(\sqrt[2]{16}\right)^3$

$= 4^3$

$= 64$

OR

$16^{\frac{3}{2}}$ 3 (EXP.), 2 (INDEX)

$= \sqrt[2]{16^3}$

$= \sqrt{4096}$

$= 64$

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10. POWERS WITH NEGATIVE EXPONENTS:

$$x^{-n} = \frac{1}{x^n} \quad \left(\frac{1}{x}\right)^n \quad AND \quad \frac{1}{x^{-n}} = x^n$$

EX.: 4^{-2}

$= \frac{1}{4^2}$

$= \frac{1}{16}$

EX.: $\frac{1}{5^{-2}}$

$= 5^2$

$= 25$

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VOCABULARY:

1. RECIPROCAL: Two numbers whose product is 1.

$5 \to \frac{1}{5}$

EX.: 2 and ½ are reciprocals.

$5\left(\frac{1}{5}\right) = \frac{5}{5}$

$= 1$

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We build on our understanding of powers to work with negative exponents.

For example:

$5^{-2} \cdot 5^{2}$
$= 5^{-2+2}$
$= 5^{0}$
$= 1$

$5^{-2} \cdot 5^{2}$

$\frac{1}{5^2} \cdot 5^2$

$\frac{5^2}{5^2}$

$= 1$

This means that 5^{-2} and 5^{2} are RECIPROCALS!
(Their product equals 1...)

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If...

$5^{-2} \cdot 5^{2} = 1$

... then...

$5^{-2} \cdot 25 = 1$

... and this must actually mean...

$\frac{1}{25} \cdot 25 = 1$

... so...

5^{-2} must be equal to $\frac{1}{25}$ or $\frac{1}{5^2}$!!!

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Another scenario based on exponent laws:

$$5^{-2} \cdot \frac{1}{5^{-2}}$$
$$= \frac{5^{-2}}{5^{-2}}$$
$$= 5^{-2-(-2)}$$
$$= 5^{-2+2}$$
$$= 5^{0}$$
$$= 1$$

This means that 5^{-2} and $\frac{1}{5^{-2}}$ are also RECIPROCALS!

(Their product also equals 1...)

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EXAMPLE:

a) 3^{-2}

$\frac{1}{3^2}$

$\frac{1}{9}$

b) 0.3^{-4}

$= \left(\frac{3}{10}\right)^{-4}$

$= \left(\frac{10}{3}\right)^{4}$

$= \frac{10\,000}{81}$

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Basically, remember to take the reciprocal of the ENTIRE base and change the negative exponent to a positive exponent.

EX.: $\left(-\frac{3}{4}\right)^{-3}$ $= \left(-\frac{4}{3}\right)^{3}$

$= -\frac{64}{27}$

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YOU TRY!

Evaluate each power.

a) 7^{-2}

$\left(\frac{1}{7}\right)^{2}$

$\frac{1}{49}$

b) $\left(\frac{10}{3}\right)^{-3}$

$= \left(\frac{3}{10}\right)^{3}$

$= \frac{27}{1000}$

c) $(-1.5)^{-3}$

$\left(-\frac{3}{2}\right)^{-3}$

$\left(-\frac{2}{3}\right)^{3}$

$= -\frac{8}{27}$

$= -0.\overline{296}$

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EXAMPLE:

Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}}$ $\left(\frac{1}{8}\right)^{\frac{2}{3}}$

$= \frac{1}{4}$

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

$= \left(\frac{16}{9}\right)^{\frac{3}{2}}$

$= \frac{64}{27}$

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YOU TRY!

Evaluate each power without using a calculator.

a) $16^{-\frac{5}{4}}$

$\left(\frac{1}{16}\right)^{\frac{5}{4}}$

$\frac{1}{32}$

b) $\left(\frac{25}{36}\right)^{-\frac{1}{2}} = \left(\frac{36}{25}\right)^{\frac{1}{2}}$

$= \frac{6}{5}$

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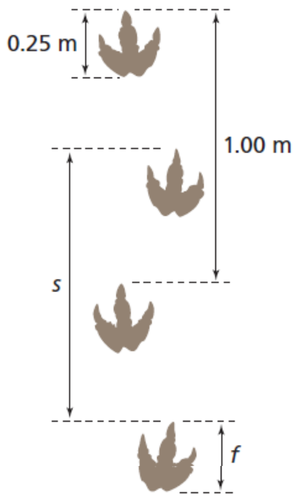
EXAMPLE:

Paleontologists use measurements from fossilized dinosaur tracks and the formula $v = 0.155\, s^{\frac{5}{3}} f^{-\frac{7}{6}}$ to estimate the speed at which the dinosaur travelled. In the formula, v is the speed in metres per second, s is the distance between successive footprints of the same foot, and f is the foot length in metres.
Use the measurements in the diagram to estimate the speed of the dinosaur.



SOLUTION

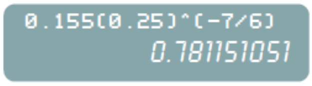
Use the formula: $v = 0.155\, s^{\frac{5}{3}} f^{-\frac{7}{6}}$
Substitute: $s = 1$ and $f = 0.25$

$v = 0.155\,(1)^{\frac{5}{3}}\,(0.25)^{-\frac{7}{6}}$

$v = 0.155\,(0.25)^{-\frac{7}{6}}$

$v = 0.7811\ldots$

The dinosaur travelled at approximately 0.8 m/s.



YOU TRY!

Use the formula $v = 0.155\, s^{\frac{5}{3}} f^{-\frac{7}{6}}$ to estimate the speed of a dinosaur when $s = 1.5$ and $f = 0.3$.

$v = 0.155\,(1.5)^{\frac{5}{3}}\,(0.3)^{-\frac{7}{6}}$

$= 1.2$ m/s

Answer: approximately 1.2 m/s

CONCEPT REINFORCEMENT:

FPCM 10:

Page 233: #3 <u>TO</u> #14

Page 234: #15 <u>TO</u> #17ab and #18 <u>TO</u> #20

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QUIZ PREPARATION - SECTIONS 4.4 & 4.5: (Fractional Exponents and Radicals; Negative Exponents and Reciprocals)

FPCM 10:

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